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AN ELEMENTARY TREATISE ON ARITHMETIC, IN THEORY AND PRACTICE:

ADAPTED TO THE INSTRUCTION OF YOUTH IN SCHOOLS AND ACADEMIES IN THE UNITED STATES.

BY JAMES RYAN,

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ADVERTISEMENT.

As UTILITY is the great object of the following Treatise, I have spared no pains to make a careful selection of materials from the most approved sources, which may tend to elucidate, in a full and clear manner, whatever is useful in the Elements of Arithmetic, both in theory and practice.

Those authors, of whose labours I have principally availed myself, are *Walker's Philosophy of Arithmetic*, and *Professor Thomson's Treatise on Arithmetic.*

The definitions and rules which it may, perhaps, be proper for the learner to commit to memory, are in the largest type employed in the work. The examples and exercises, and the principal illustrations, are in a character somewhat smaller; and the notes at the bottom of the pages, being in a type still smaller, contain several important illustrations and remarks, which may often be very interesting to those students who have made some progress in the science of Arithmetic.

The reasons of the rules and operations are explained, not in strict, formal demonstrations, but generally by simple and easy illustrations of particular cases and examples; and it is hoped that the subject will thus be rendered as intelligible and attractive as possible. This part of Arithmetic, as has been very justly observed by *Professor Thomson*, is too generally neglected in treatises on this subject, and thus one of the principal divisions of Mathematical Science is converted into a mere practical art, and what is calculated to call forth and improve the reasoning power of the pupil, is degraded into a dry exercise of memory.

Of the examples and exercises, some are proposed in purely abstract terms, being intended merely to afford practice to the learner in the rules and modes of calculation. To these are subjoined, in those parts of the work in which it could be conveniently done, other questions, which will not only afford

the pupil further exercise on the rules which precede them, but will also furnish him with many important facts in Commerce, Geography, Astronomy, Chronology, Chemistry, and other departments of knowledge.

As the information contained in these questions has been all derived from authentic sources, its correctness may be depended on; and, it is hoped, that what is thus presented, may excite, in the young reader, a desire to enrich his mind by the acquisition of further information of a similar nature.

J. R.

NEW-YORK, May 15th, 1827.

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AN ELEMENTARY

TREATISE ON ARITHMETIC.

CHAPTER I.

NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF WHOLE NUMBERS.

1. WHATEVER is capable of increase or diminution is, in general, called *magnitude*, or *quantity*.

For example, a sum of money is a quantity, since we may increase or diminish it. In like manner, a weight and other things of this nature are quantities.

2. In order to measure or determine any quantity, we must consider some other quantity of the same kind as known, which shall be used as a term of comparison: and this quantity is called a *unit*.

If it were proposed, for example, to determine the quantity of a sum of money, we should take some known piece of money, as a dollar, an eagle, a crown, or some other coin, and show how many of these pieces are contained in the given sum. In the same manner, if it were proposed to determine the quantity of a weight, we should take a certain known weight, for example, a pound, an ounce, &c. and then show how many times one of these weights is contained in that which we are endeavouring to ascertain. If we wished to measure any length or extension, we should make use of some known length, such as a foot. So that the determination, or the measure of magnitude of all kinds, is reduced to this: fix at pleasure upon any one known magnitude of the same species with that which is to be determined, and consider it as the *measure*, or *unit*; then determine the mutual relation of the proposed magnitude and this measure.

This relation is always expressed by numbers; so
number is nothing but the relation of one magni-
to another, arbitrarily assumed as the unit.
combination of *units* gives rise to *whole numbers*;
manner of forming, expressing, and writing numbers
means of characters invented for that purpose, is
e object of *numeration*; and the science which teaches
ow to perform different operations upon numbers is
called *Arithmetic*.

From this it appears that all magnitudes may be expressed by numbers; and that the foundation of all the mathematical sciences must be laid in a complete treatise on the science of numbers, and in an accurate examination of the different possible methods of calculation.

Numeration of Whole Numbers.

4. In order *to form whole numbers*, we must begin with *unity*, or *one*; unity added to itself gives a number named *two*; this increased by one composes the number *three*; and by adding successively unity to each
mber obtained, we compose the numbers *four*, *five*,
seven, *eight*, *nine*. This last increased by unity
he number *ten*; the collection of ten units form
der of units, named *tens*.
same manner as we have counted from unity
ve shall also count from one ten to nine tens;
e words one ten, two tens, three tens, four
tens, seven tens, eight tens, nine tens,
hirty, *forty*, *fifty*, *sixty*, *seventy*, *eighty*,
numbers comprised between two
s successively the tens and the
three tens and seven units is
stem, the numbers com-
r, instead of the words
leven, *twelve*, *thirteen*,
n, *nineteen*.
ne units forms
a *unit* gives

the number hundred, and the collection of ten tens is called *hundred*. We reckon from one hundred to nine hundred, and in order to designate the numbers comprised between two consecutive hundreds, we add to the names one hundred, two hundred, three hundred, &c. those of the first ninety-nine numbers. We arrive thus at the number nine hundred and ninety-nine; this increased by unity or one gives ten hundreds; this collection of ten hundreds is called *thousand*; and, in like manner, as we have counted by units, tens, and hundreds of units, from one unit to a thousand units, we count by units, tens, and hundreds of thousands, from one unit of a thousand to one thousand units of a thousand, called *million*.

6. The number nine hundred and ninety-nine thousand nine hundred and ninety-nine increased by unity or one, gives a collection of thousand of thousands, or a million; a thousand millions form a *billion*; and so on.

According to this system, the name of a number depends only upon the names of the first nine hundred and ninety-nine numbers, with the words, *unity*, *thousand*, *million*, *billion*, &c.; so that *all numbers may be expressed by means of the nine units, nine tens, and nine hundreds of each kind*. The units, the tens, the hundreds, the thousands, &c. are named also *units of the first order*, *of the second order*, *of the third order*, *of the fourth order*, &c.; and the simple units, the thousands, the millions, &c. are units of a *ternary order*.

7. The simplicity of the numeration language, resulting from the small number of words necessary to express all numbers, has furnished the idea of writing them by the aid of some *characters*, called *figures*. Thus, as we have invented nine names to express the first nine numbers, we adopt nine figures to represent them; and as the combination of these nine names, with those of the different orders of units, have given the names of all the numbers, we are convinced that the figures, placed in a line, (one after the other,) would indicate by their values the number of units of each

kind, and, by their position, the order of these units. These nine figures are—

1, 2, 3, 4, 5, 6, 7, 8, 9.

They represent the numbers

one, two, three, four, five, six, seven, eight, nine.

8. In order to write a number composed of units, tens, hundreds, &c. we must place the figures which indicate the number of units of each order in such a manner that the figure of the units, or of the first order, shall occupy the first rank to the right hand side; that of the tens, or of the second order, the second rank, &c.

Thus, the number nine thousand five hundred and sixty-seven is written 9567.

9. Several numbers cannot be written by this system of *numeration;* we could not, by its means, write a number which does not contain the units of all the inferior orders to its highest unit. To remedy this inconvenience, we have invented the auxiliary figure 0, named *cipher*, or *zero*, which, having no value whatever by itself, serves to preserve all the *significant figures*, 1, 2, 3, 4, 5, 6, 7, 8, 9, in the rank which agrees to the order of their units.

Thus, in order to write the number nine hundred and seven, we put zero between 9 and 7, in order to occupy the place of tens, and thus to make the other figures 9 and 7 occupy those places in which they will express the intended values; and the number is written in figures thus, 907. Thus, by the help of the nine significant figures and zero, which is used to fill places where no value is to be expressed, we are able to designate all numbers, however great; and this, while each of the figures (sometimes called *digits*, from the Latin word signifying a finger,) always retains the same numeral significancy. For example, in the two numbers 57 and 570, the character 5 denotes in each the num-
r five, and the character 7 the number seven: but in the
er, the 5 standing in the second column designates fifty;
the latter, where it stands in the third column, it de-
hundred: and in the former, the 7 standing in

the right hand column designates seven units; but in the latter, standing in the second column, it designates seven tens, or seventy. And thus we see that the cipher, though it denotes that there is *no* number belonging to its column, must still be written, in order to bring the significant figures into their proper places. If, therefore, we want to express the number *four million and sixty-eight thousand and fifty-three*, the seventh column being the place of millions, the character 4 must be followed by six figures; and the fourth column being the place of thousands, the characters 68 must be followed by three figures: and thence we conclude that besides the significant figures 4, 68, and 53, zero must be interposed between the latter two, and another zero between the former two: thus, 4068053.

10. To facilitate numeration, we commonly mark off by a comma every period of three figures, commencing from the right hand. And as the name of a *million* is given to ten hundred thousand, so the number ten hundred (or a thousand) millions is called a *billion*. In like manner, the names of a *trillion*, *quadrillion*, &c. are given to a thousand billions, trillions, &c.

But here it is to be observed, that the facility with which we can designate the highest numbers, and perform every arithmetical calculation on them, has occasioned an insensibility to the enormous magnitude of the numbers of which we speak. One billion is very easily mentioned, and easily designated by a unit followed by nine ciphers: thus, 1,000,000,000. The following consideration may, perhaps, assist in enlarging the ideas of the pupil on this subject: to count a billion, at one per second, would require nearly thirty-two years.

For this method employed for designating numbers by the aid of written characters, as well as some other important improvements in arithmetic, we are indebted to the Arabs. It was brought by the Moors into Spain; and John of Basingstoke is supposed to have introduced it into England, about the middle of the 11th century.

The Roman method of notation prevailed in Europe before the introduction of the Arabic; and the Greeks employed a numeral notation similar to the Roman.

11. The names of the local values of figures, &c.

cording to the Arabic Notation, will be known from the following table.

NUMERATION TABLE.

I. Units	1 Units, 2 Tens, 3 Hundreds
II. Thousands	4 Units, 5 Tens, 6 Hundreds
III. Millions	7 Units, 8 Tens, 9 Hundreds
IV. Billions	10 Units, 11 Tens, 12 Hundreds
V. Trillions	13 Units, 14 Tens, 15 Hundreds
VI. Quadrillions	16 Units, 17 Tens, 18 Hundreds
&c.	&c.

From this table, it appears that if a line of figures be divided into periods of three figures each, commencing at the right hand, the first period will contain units, the second thousands, the third millions, &c.; and it is usual and convenient thus to divide the figures by which large numbers are expressed, for the purpose of facilitating their numeration. The periods succeeding those contained in the table are *quintillions*, *sextillions*, *septillions*, *octillions*, and *nonillions;* and analogical names might be formed for the still higher periods. Those already given, however, are more than sufficient to express any number which it is ever necessary to designate in language. The local value of any figure used in expressing a number is at once discovered from this table: thus, 6 in the eighth place from the right hand expresses six tens of millions, or sixty millions; and, conversely, sixty millions will be expressed by the figure 6 in the eighth place.*

* This method of dividing lines of figures into periods, and of naming those periods, as has been very properly observed by *Professor Thomson*, is that which is used by the French and Italians. It is strongly recommended by its simplicity and elegance; and has been adopted in

12. From the view we have taken of the Arabic notation, it is plain that zero, wherever it occurs, increases tenfold the value of every figure standing on its left hand, but does not affect the value of the figures standing on its right hand.

It may be observed, that instead of counting tens and combinations of tens, we might as well count by twelves and combinations of twelves; or by any other number sufficiently low. And to the numeration by twelves, for instance, a notation similar to the Arabic may be applied, only introducing two new characters to designate the numbers *ten* and *eleven*. Then the figure 10 would denote the number *twelve;* for the 1, standing in the second column, would de-

the treatise on Arithmetic by *Mr. Anderson*, in the *Edinburgh Encyclopedia;* and in *Professor Leslie's Philosophy of Arithmetic:* see *Professor Thomson's Treatise on Arithmetic, in Theory and Practice.*

In other English and American works, the periods are made to consist of six each, and have the same names as those in the table given above, except thousands, for which there is not a distinct period. The two methods agree as far as hundreds of millions, and it is rarely necessary to name larger numbers.

In the ancient Roman notation, I. signified one, V. five, X. ten, L. fifty, and C. one hundred. To these were added, at a later period, D. signifying five hundred, and M. one thousand. Where any character was followed by another, of equal or of less value, the compound value was equal to the simple value of *both taken together;* but when a character preceded one of greater value, both together expressed a value equal to the *difference* of their simple values. Thus, II. expressed two, XI. eleven, and IX. nine; CX. one hundred and ten, XC. ninety. We find also IↃ put for 500; and by every such Ↄ, the value is made ten times as great. Thus, IↃↃ. signifies 5,000, IↃↃↃ. 50,000, &c. CIↃ. was also used to express 1000, and the prefixing C. and the annexing of Ↄ. increased its value ten times. Thus, CCIↃↃ signified 10,000, &c. A line drawn over the top of a letter made it signify as many thousands as the letter itself expressed units: thus, $\overline{\text{V}}$. expressed 5000, $\overline{\text{C}}$. 100,000, &c. The following table, together with the preceding observations, will give an adequate idea of the Roman notation.

I	1	IX	9	LXXX	80
II	2	X	10	XC	90
III	3	XX	20	C	100
IV, or IIII	4	XXX	30	D, or IↃ	500
V	5	XL	40	M, or CIↃ	1000
VI	6	L	50	MM, or $\overline{\text{II}}$	2000
VII	7	LX	60	IↃↃ, or $\overline{\text{V}}$	5000
VIII	8	LXX	70		

MDCCCXXVII, or CIↃIↃCCCXXVII, 1827.

note one parcel of twelve: and the figures 203 would denote the number two hundred and ninety-one; for the 2, standing in the third column, would denote two parcels of twelve times twelve each, that is, two hundred and eighty-eight. And certainly if this *duodecimal* notation had been originally adopted, and the language accommodated to it by affording distinct names for the several combinations of twelve, it would have possessed a considerable advantage over the *decimal* notation, which proceeds by combinations of tens. For the number twelve admitting four divisors, (namely, 2, 3, 4, 6,) while the number ten can be evenly divided only by 2 and 5, we would be much less frequently involved in fractional remainders than at present. And if all the divisions of measures, weights, coins, &c. ran in the same duodecimal progression, the practical advantages would be great.

But it appears from the structure of all known languages, that numeration by *tens* has been adopted by all nations in all ages, rather than the numeration by twelves, or any other number. And this is obviously to be accounted for from the natural circumstance of the number of our fingers; the fingers being, in the origin of society, the readiest instrument to assist numeration; and still, indeed, frequently employed for that purpose by the illiterate peasantry. So that we may conclude, that if nature had furnished man with twelve fingers instead of ten, the duodecimal numeration would have been as general as the decimal now is; and languages would have abounded as much with names for the combinations of twelve, as they now do with names for the combinations of ten.

To express Numbers by Figures, according to the Arabic Notation.

13. Rule.—Make a sufficient number of ciphers or dots, and divide them into periods; then, commencing at the left, place in their proper positions, beneath the ciphers or dots, the significant figures necessary for expressing the proposed number. If any places remain unoccupied, let them be filled with ciphers.

Thus, the method of expressing the number two hundred

and five millions, twenty thousand, seven hundred and nine, will be found in the following manner:

000,000,000 . . .,. . .,. . .
2 5 2 · 7 9, or 2 5 2 7 9

and thence, by filling the unoccupied places, 205,020,709. By practice, the learner will soon be enabled, in most cases, to dispense with the ciphers or dots.

Exercises in Notation.

Express the following numbers in figures:—

Ex. 1. Fifty-two.
2. Three hundred and fifteen.
3. Four hundred and five.
4. One thousand three hundred and four.
5. Seven thousand and eighty-four.
6. Nine thousand and nine.
7. Six thousand and seventy.
8. Twenty thousand and seventy-three.
9. Four hundred and six thousand and nine.
10. Six hundred and fifty thousand and ninety.
11. Seven millions seven thousand and ten.
12. Twenty-five millions three hundred thousand.
13. Eleven millions two hundred and ten.
14. One hundred and ten millions and twenty thousand.
15. One billion ten millions two thousand.
16. One trillion one hundred billions two millions.
17. One quadrillion and nineteen millions.
18. Nine hundred billions six millions and five.
19. The world was created two thousand three hundred and forty-eight years before the Deluge; three thousand two hundred and fifty-one years before the building of Rome; four thousand and four years before the birth of Christ; five thousand four hundred and ninety-six years before the discovery of America; and five thousand eight hundred and thirty-one years before the present time (1827.) Let each of these numbers be expressed in figures.
20. The following numbers express the distances of the primary planets from the sun, in American miles; express them in figures.—Mercury, thirty-seven millions; Venus, sixty-nine millions; the Earth, ninety-five millions; Mars, one hundred and forty-five millions; Vesta, two hundred and twenty-five millions; Juno, two hundred and fifty-three

millions; Ceres, two hundred and sixty-two millions seven hundred and fifty thousand; Pallas, two hundred and sixty-three millions; Jupiter, four hundred and ninety-four millions; Saturn, nine hundred and six millions; Uranus, or Herschel, one billion eight hundred and twenty-two millions.

To express in Words the Numbers denoted by Lines of Figures.

14. Rule.—Commencing at the right hand, divide the given figures into periods of three figures each, till not more than three remain. Then the first period towards the right hand contains units or ones, the second thousands, the third millions, &c. as in the Numeration Table: and, therefore, commencing at the left side, annex to the value expressed by the figures of each period, except that of the units, the name of the period.

Thus, the expression 37053907 becomes, by division into periods, 37,053,907, and is read *thirty-seven millions fifty-three thousand nine hundred* and *seven*, the term *units* or *ones*, at the last, being omitted. By practice, the pupil will soon find it unnecessary to divide into periods any lines of figures except those of considerable magnitude.

Exercises in Numeration.

Write down in words, or name, the numbers signified by the following expressions:—

Ex. 1.	88	Ex. 11.	30689
2.	225	12.	107000
3.	365	13.	185000
4.	687	14.	170000
5.	1335	15.	217000
6.	1591	16.	303000
7.	1681	17.	704000
8.	1682	18.	2050000
9.	4333	19.	327437913
10.	10759	20.	198404567

Ex. 21. 262921240
22. 64516673
23. 402265155
24. 23810000
25. 906183000

Ex. 26. 101023456789
27. 102030405060708090
28. 1000100010010 1
29. 9000103456078
30. 7610345634567890000

Questions.

What is magnitude or quantity?

What is the term of comparison called, by which we measure any quantity?

What is a number? And what gives rise to whole numbers?

What is numeration?

What is arithmetic?

How are whole numbers formed?

How is the number one hundred formed?

How many hundreds compose a thousand?

How many thousands compose a million?

How many millions form a billion?

How many figures are usually necessary to express numbers?

What are their names?

How do you write a number composed of units, tens, hundreds, &c. by means of figures?

Can all numbers be written by means of nine figures?

What other figure, besides those nine, is necessary to express all numbers?

What is the use of the character cipher or zero?

What is the rule for expressing numbers by figures?

How do you read in words a number denoted by a line of figures?

Addition of Whole Numbers.

15. The operation by which we express the total value or amount of several given numbers in one *sum*, is called *addition of whole numbers.*

For example, six dollars and nine dollars, expressed in one sum, are fifteen dollars.

To perform the operation of addition, it is necessary that the learner should be able to assign the sum of any two given numbers not exceeding nine; and for this purpose he shoul be exercised in the following table.

Addition Table.

2 and	2 and	3 and	4 and	5 and	6 and	7 and
2=4	9=11	7=10	6=10	6=11	7=13	9=16
3=5	—	8=11	7=11	7=12	8=14	—
4=6	3 and	9=12	8=12	8=13	9=15	8 and
5=7	3=6	—	9=13	9=14	—	8=16
6=8	4=7	4 and	—	—	7 and	9=17
7=9	5=8	4=8	5 and	6 and	7=14	9 and
8=10	6=9	5=9	5=10	6=12	8=15	9=18

To enable the learner to acquire accuracy and despatch in addition, it is proper to train him to add in the following manner, till he can do it with facility. Since 6 and 6 are 12, 26 and 6 are 32; (here it should be pointed out to him that 12 and 32 end in the same figure;) since 9 and 7 are 16, 39 and 7 are 46; since 8 and 6 are 14, 88 and 6 are 94; since 9 and 6 are 15, 9 and 16 are 25; since 8 and 9 are 17, 8 and 99 are 107, &c.

16. In addition, we successively take the sum of the digits standing in each column, and combine those sums into one total.

The reason of commencing from the right hand column, or place of units, and proceeding from right to left, is, that we may carry on the combination of the sums of the several columns as we proceed. Thus, in adding together 509 and 293, the sums of the numbers standing in the several columns are 12 units, 9 tens, or 90, and 7 hundreds, or 700. Now, adding the 1 ten contained in the 12 units to the 9 tens, (the sum of the second column,) we have 10 tens, or 1 hundred; which added to the 7 hundreds, (the sum of the third column,) gives 8 hundreds; and these combined with the 2 units in the sum of the first column give 802 as the total. By proceeding from right to left, we are saved the trouble of writing the several sums of the several columns separately, and afterwards combining them by a second addition. We write down under each column the right hand figure of its sum, and *carry* the other figures to the next column. But the same result will be obtained by repeated additions, proceeding from left to right, or taking the ums of the columns in any order. And in this way, the ung scholar may be made to prove his work.

In arranging the numbers which we want to add, it is obviously needful that the digits of the corresponding columns of each number should be disposed in lines exactly under each other: and the scholar ought to be exercised in the due arrangement of the numbers for himself, and not have them given him arranged by the teacher.

The sign +, interposed between two numbers, denotes that the numbers are to be added: this sign is technically called *plus*, from the Latin word signifying more. Thus, 23+5 (read 23 *plus* 5) denotes the sum of 23 and 5.

To add Whole Numbers.

17. RULE.—Place the numbers so that units may stand under units, tens under tens, &c. Find the sum of the column of units, set down the last figure of it below that column, and carry to the next the number expressed by the remaining figure or figures, if there be any. Proceed as before with the remaining columns, and at the last column set down the entire sum.

Thus, to add together 9468, 2956, and 79, let them be set as in the margin: then, 9 and 6 are 15 and 8 are 23, set down 3 and carry 2 to the column of tens. Then, 2 and 7 are 9 and 5 are 14 and 6 are 20, set down a cipher and carry 2; again, 2 and 9 are 11 and 4 are 15, set down 5 and carry 1; finally, 1 and 2 are 3 and 9 are 12, set down 12; and the sum or answer required is 12503; that is, twelve thousand five hundred and three.

```
 9468
 2956
   79
-----
12503, sum.
-----
```

Methods of Proof.

1. Add the several columns, according to the rule, commencing at the top and proceeding downward, and if the result be the same as was obtained by adding them upward, it may be presumed that the work is right.

2. Separate the given numbers into two or more divisions. Find the sum of these divisions severally, and add these partial sums together. If the last result be equal to that found by the common method, the work is right.

This will appear obvious from the following example:

	37928
	93640
	23574
	75849
Entire sum	230991
Sum of the first division	131568
second	99423
Entire sum	230991, proof.

This method may also be employed with advantage in finding the sums of large columns, instead of adding the whole at a single operation. The first method is very convenient and useful, when the columns are not very large.

3. Commencing at the left hand, add the several columns, without *carrying*, and set down the full sum of each column, with the units in their proper place, and the tens below the figure immediately to the left. Add together the two lines thus resulting, and if the last result agree with that obtained by the common method, it may be concluded that both are right.

Thus, in the annexed example, the sum of the left hand column is 25, which is set down in full: the sum in the next column is 30; the cipher is set in its proper place, and 3 under the 5; and so with the rest. The sum of the two lines thus obtained is equal to the sum found by the ordinary method.*

5946
9738
2697
9868
28249, sum.
25029
322
28249, proof.

This method of addition might be used instead of the common method; and as it requires nothing to be carried, it might be employed with advantage when the calculator is liable to interruptions.

* Addition may also be proved by *casting the nines* out of each of the given numbers, as will be explained in multiplication; and by casting the nines out of the sum of the excesses, and out of the sum of the numbers. If these latter excesses be equal, the work is generally right; otherwise, it *must* be wrong. This method, however, is of little use in proving addition.

Exercises in Addition of Whole Numbers.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.	Ex. 5.
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	5	6

Ex. 6.	Ex. 7.	Ex. 8.	Ex. 9.	Ex. 10.
7	8	9	1	11
7	8	9	2	11
7	8	9	3	11
7	8	9	4	11
7	8	9	5	11
7	8	9	6	11
7	8	9	7	11
7	8	9	8	11
7	8	9	9	11
7	8	9	10	11

Ex. 11.	Ex. 12.	Ex. 13.
756	450	503
123	39	471
345	100	99
567	7	123
789	830	398
901	789	17

Ex. 14.	Ex. 15.
7304567	123456789
23451	987654321
120009	90
2030970	100000009

16. Required the sum of 37545+23014+756+830 +756+879109.

17. Required the sum of 120350+819+365+7569 +1827+10734.

18. Required the sum of 12756+10019+756897+ 13569+8769.

19. Required the sum of 976+3456+897436+ 123456+897030+1978394+7568934.

20. Add together 376454, 1276003, 756000, 376976, 701000134, 37564, and 75603.

21. Add together seven thousand nine hundred and seventy-six, eight thousand six hundred and twenty-four, one million one thousand and thirty, three hundred and sixty-five, one thousand seven hundred and twenty-seven.

22. The following are the reigns of the Saxon kings of England, from the union of the kingdoms of the Heptarchy under Egbert, to the Danish conquest.

Egbert reigned -	14 years	Edred reigned - -	8 years
Ethelwolf - - -	20	Edwy - - - - -	4
Ethelbald - - -	3	Edgar - - - - -	15
Ethelbert - - -	6	Edward II. - - -	4
Ethelred - - -	6	Ethelred II. - -	37
Alfred - - - -	29	Edmund (Ironside)	1
Edward I. - - -	23		
Athelstan - - -	16		
Edmund I. - -	7		

Required the sum of these years.

23. Egbert began to reign in 823; when did Alfred's reign commence? Ans. 872.

24. When did Edwy commence his reign? Ans. 955.

25. The world was created 4004 years before Christ; how long is it since? Ans. 5831.

26. Required the aggregate population of the following states:—

That of Maine, in 1820,	297839
New-Hampshire	244155
Vermont	235764
Massachusetts	521725
Rhode Island	83059
Connecticut	275248
Ans.	

27. Required the aggregate population of the following states:—

The population of	New-York	1372812
	New-Jersey	277575
	Pennsylvania	1049458
	Delaware	72749
	Maryland	407350
	Virginia	1065304
	North Carolina	638829
	South Carolina	490309
	Georgia	340989
	Ans.	

28. The population of Florida is 10000

Alabama	143000
Mississippi	75448
Louisiana	153407
Tennessee	422813
Kentucky	564313
Illinois	55211
Indiana	147178
Ohio	581295
Arkansaw	14273
Missouri	66586

Required the sum. Ans.

29. The population of America is 40000000

Europe	190000000
Asia	340000000
Africa	70000000
Austral-Asia	20000000

Required the sum. Ans.

30. Sir Isaac Newton, the celebrated mathematician, was born in the year 1642, and died in his eighty-fifth year. In what year did he die? Ans.

31. George Washington was unanimously elected President of the United States in 1789, and presided 8 years; John Adams presided 4 years; Thomas Jefferson 8 years; James Madison 8 years; James Monroe 8 years. In what year was the last President's time expired? Ans.

32. In 1821, the population of the following towns in

England, Scotland, Ireland, and France, (the three largest in each,) was as follows:—

London - - - -	1274800	Glasgow - - -	147043
Manchester - -	133788	Edinburgh - - -	138235
Liverpool - - -	118972	Paisley - - - -	47003
Dublin - - - - -	186276	Paris - - - -	720000
Cork - - - -	100535	Lyons - - - -	115000
Limerick - - - -	66042	Marseilles - - -	102000

Required the number of inhabitants contained in the three largest towns in each country. Ans.

33. The Pyramids of Egypt are thought to have been built 337 years before the founding of Carthage, Carthage to have been founded 49 years before the destruction of Troy, and Troy to have been destroyed 431 years before Rome was founded; Carthage was destroyed 607 years after the founding of Rome, and 146 years before the commencement of the Christian era; the western empire of Rome ended in the year 476 of the Christian era, and 590 years before the Norman conquest; Constantinople was taken by the Turks 387 years after the Norman conquest, and 323 years before the American Revolution. How many years elapsed between the first and last of these events?

Questions.

What is addition?
How is addition performed?
Repeat the rule of operation.
How is addition proved?

Subtraction of Whole Numbers.

18. Subtraction of whole numbers is the taking of a less number from a greater, and thereby showing the remainder or difference.

The number which is to be subtracted from the other is usually called the *subtrahend;* the number from which the subtraction is made, the *minuend.* If we have to subtract 346 from 579, it is plain that we may subtract the units, tens, and hundreds of the minuend; and that the sum of the remainders 233 is the remainder sought. And in such a case, it matters not whether we proceed from left to right, or from right to left. But if any digit of the minuend be less

than the digit in the corresponding column of the subtrahend, for instance, if we have to subtract 279 from 546, as we cannot subtract 9 units from 6 units, nor 7 tens from 4 tens, we may suppose the minuend resolved into the parts 16, 130, and 400; and then subtract the 9 units from 16, the 7 tens from 13 tens, and the 2 hundred from 4 hundred. And thus, when any digit of the minuend is less than the corresponding digit of the subtrahend, conceiving a unit prefixed to it, and performing the subtraction, when we proceed to the next column we have to conceive the next digit of the minuend less by 1, on account of the 1 which has been already borrowed from it. But it affords the same result, in practice, to conceive the next digit of the subtrahend increased by one, and the digit of the minuend unaltered: as it obviously gives the same remainder to subtract 8 from 14, as to subtract 7 from 13. And hence appears the reason of what is called *carriage*, in subtraction; and the reason of proceeding from right to left: though the result may be obtained by repeated subtractions from left to right. The carriage in subtraction may be accounted for on another principle; namely, that if the two numbers be equally increased, their difference will remain unvaried. Thus, in subtracting 19 from 56, when we take 9 from 16, we may conceive that we have added 10 to the minuend, and therefore must add 10 also to the subtrahend.

Besides the same attention to the arrangement of the numbers as is necessary in addition, the scholar ought to be exercised in performing the operation of subtraction whether the subtrahend be above or below the minuend.

The remainder found being the difference between the given numbers, or the numbers by which the minuend exceeds the subtrahend, it is plain that adding the remainder to the subtrahend must give a total equal to the minuend: or that subtracting the remainder from the minuend must give a remainder equal to the subtrahend. This affords two methods of proving subtraction. And, in addition, if we subtract any one of the numbers from the total, the remainder must be equal to the sum of all the other numbers.

It is proper to observe, that the sign — (called *minus*, from the Latin word signifying less,) interposed between two numbers, denotes that the latter is to be subtracted from the former: thus, 25—4 (read 25 *minus* 4) denotes the remainder 21, subtracting 4 from 25.

Rule for the Subtraction of Whole Numbers.

19. Place the less number below the greater, with units under units, tens under tens, &c. as in addition. Beginning with the units, take, if possible, each figure in the lower line from the figure above it, and set down the remainder. But if any figure in the lower line be greater than the figure above it, add ten to the upper; then subtract as before, and carry one to the next figure in the lower line.

The reason of this rule is evident from the preceding article.

Methods of Proof.

1. Add the remainder and the less of the given numbers together: if the sum be equal to the greater, the work is correct.

2. Subtract the number found from the greater of the given numbers; if the remainder be equal to the less, the work is correct.

Examples in Subtraction of Whole Numbers.

Ex. 1. From 7854 take 4513.

Set the numbers as in the margin, and proceed thus: 3 from 4 and 1 remains, 1 from 5 and 4 remain, 5 from 8 and 3 remain, 4 from 7 and 3 remain; the remainder therefore is 3341.

To prove the work, to the less of the given numbers add the remainder, and the sum will be 7854, the greater; or, as in the second method, subtract the remainder from the greater number, and the result will be 4513, the less.

	7854
	4513
Rem.	3341
Proof	7854

	7854
	4513
Rem.	3341
Proof	4513

Ex. 2. Required the difference of 3712 and 1831.

In this example, proceed thus: 1 from 2 and 1 remains, 3 from 11 and 8 remain, carry 1 to 8, and then 9 from 17 and 8 remain, carry 1, and then 2 from 3 and 1 remains. The differ-

From	3712
Take	1831
Rem.	1881

ence therefore is 1881, and the operation would be proved in the same manner as before.

Ex. 3. Required the difference of 83 and 57.

Here, as 7 cannot be taken from 3, we consider 83 as 70 and 13; and, subtracting 7 from 13, and 5 from 7, we find the difference to be 26. In this simple and natural method, the values of the given numbers undergo no change; and, with only one exception, it might be employed with as much facility as the common method, the next figure in the upper line being always diminished by a unit, when one would be carried to the figure below it, in the common method. The exception is the case in which the next figure in the upper line is a cipher; in this case, the common method is considerably preferable; and, as in practice that method is in no way inferior, it is universally preferred.

```
83
57
——
26
```

Whenever the figure in the lower line is greater than the corresponding figure in the upper line, the operation of subtraction may also be performed in the following manner:—subtract the less from the greater, and take the remainder from ten; set down the result; then take the next figure in the lower line from the corresponding figure in the upper line diminished by unity, and proceed as before. For instance, in the above example, as 7 cannot be taken from 3, subtract 3 from 7, and there remain 4, and 4 from 10 and 6 remain: now 8 diminished by 1 is 7, and 5 from 7 and 2 remain. By this method, we can perform the operation of subtraction without the aid of addition, and also without being under the necessity of carrying one, as in the common method.

Exercises in Subtraction of Whole Numbers.

Ex. 1.	17	Ex. 2.	49	Ex. 3.	100	Ex. 4.	10000
	8		39		1		9999

Ex. 5.	3767456	Ex. 6.	123456789
	1245632		12345678

Ex. 7.	756307—32706	Ex. 10	13450—7569
8.	130076—12780	11	1827—1492
9.	456789—356345	12.	10000—1001

Ex. 13.	1010101—101010	Ex. 17.	44444—35555
14.	10000000—1000101	18.	12222—3333
15.	75634—12345	19.	303030—30303
16.	9999—8888	20.	95000000—240000

21. Take four thousand four hundred and four from four millions.

22. Required the difference between six millions and six thousand.

23. Subtract nineteen millions and ninety-nine from one billion.

24. La Place, the celebrated French mathematician and philosopher was born in 1749; required his age in 1826.

25. The height of Mont Blanc, the highest mountain in Europe, is 15680 feet, and the height of Chimborazo, the highest mountain in America, is 21427 feet: how much is the latter higher than the former?

26. Required the difference between the population of New-York and that of New-Jersey? Ans. 1095237.

27. How long is it since James Monroe was elected to the Presidency, which event took place in 1817? Ans.

28. Guns were first used in 1380: how many years since that time till the year 1827? Ans. 447 years.

20. How many inhabitants are there in Africa more than in America? Ans. 30000000.

30. George Washington was elected President of the United States in the year 1789: how many years have elapsed since? Ans.

31. How long is it since the peace of Utrecht, which was concluded in the year 1713? Ans.

32. La Fayette arrived in America in the spring of the year 1777. How many years have elapsed since? Ans.

33. The following are the years of the Christian era in which the under-mentioned events happened: required the number of years from each till the year 1827. Commencement of the Hegira, or era of the flight of Mahomet, 622; the Arabic, or modern notation in Arithmetic, introduced from Arabia into Europe by the Saracens, 991; first Crusade, 1096; Magna Charta signed by king John, 1215; linen first made in England, 1253; Termination of the Crusade, 1291; spectacles invented by a monk of Pisa, 1299; gunpowder first used in Europe, 1330; Algebra introduced into Europe from Arabia, 1412; printing invented, 1440; Constantinople taken by the Turks, 1453; America discovered

by Columbus, 1492; Vasques de Gama's discovery of the route to the East Indies by the Cape of Good Hope, 1497; commencement of the Reformation, 1517; spinning-wheel invented, 1530; Copernicus died, 1543; telescopes invented, 1590; University of Dublin founded, 1591; English East India Company established, 1600; Decimal Fractions invented, 1602; thermometers invented, and satellites of Jupiter discovered, 1610; Logarithms published by Napier, 1614; circulation of the blood discovered by Harvey, 1619; barometers invented, 1643; air-pump invented, 1654; Newtonian philosophy published, 1686; Cornwallis surrendered to the American army in 1781; the Constitution of the United States submitted to Congress, 1787; union of Great Britain and Ireland, 1801; battle of Waterloo, 1815.

Questions.

What is subtraction?
Repeat the rule for performing the operation.
What are the methods of proof.

Examples in Addition and Subtraction.

Ex. 1. A man has five apple-trees, of which the first bears 157 apples, the second 264, the third 305, the fourth 97, and the fifth 123. He sells 428 apples; 186 are stolen. How many has he left for his own use? Ans. 332.

Ex. 2. Out of an army of 57068 men, 9503 are killed in battle; 586 desert to the enemy; 4794 are taken prisoners; 1234 die of their wounds on the passage home; 850 are drowned. How many return alive to their own country? Ans. 40101.

Ex. 3. A man, travelling from New-York to Washington, went the first day 35 miles, the second day 60 miles, the third day 59 miles, and going the fourth day 36 miles, he was within 38 miles of Washington. What is the distance between New-York and Washington; and how far from the latter city was the traveller at the end of the third day?

Ans. From N. Y. to W. 228 miles, and 74 miles.

Ex. 4. A man, at the beginning of the year, finds himself worth 123078 dollars. In the course of the year, he gains by trade \$8706; but spends in January \$237, in February \$301, and in each succeeding month (being 10 months) as

much as in the first two. What was the state of his affairs at the end of the year? Ans. $125866.

Ex. 5. In the South District of the state of New-York, consisting of six counties, the population in 1820 was 219457; the population in the Middle District, consisting of nine counties, 237650; in the East District, consisting of eleven counties, 243826; and in the West District, consisting of twenty-four counties, 1372812. Required the excess of the population of the West District over the population of the other three Districts. Ans. 671879.

Ex. 6. A merchant bought 756 barrels of flour for $3560, 259 barrels for $1250; and he sold afterwards 1000 barrels for $5000. How much does he gain, and how many barrels has he left? Ans. He gains $190, and 15 barrels unsold.

Ex. 7. The public debt of the United States in 1791, was $75463467; from 1791 till 1812, the debt was reduced $38806535; from this date till 1816, it was increased $86359443; and from this date till 1820, it was reduced $31336285. What was the public debt in the year 1820? Ans. $91680090.

Ex. 8. James Cook, the navigator, was born in 1728, and he was killed in 1779. How old was he at his death? Ans. 51 years.

Ex. 9. Alexander Pope, the poet, was 55 years old when he died in 1744. In what year was he born? Ans. 1689.

Ex. 10. William Penn, the founder of Pennsylvania, landed first at New-Castle, in the year 1682: How many years since his landing till the present time (1827?) Ans. 145 years.*

Multiplication of Whole Numbers.

20. *Multiplication of whole numbers* is but an abridged method of addition, employed where we have occasion to add the same number repeatedly to itself.

Of the two numbers multiplied together, and called

* Chronology will furnish the teacher with an indefinite variety of examples; but it is to be observed, in general, that pains should be taken to give the child a clear conception of the terms employed in a question, before he is called to solve it: and that the first illustrations of the use of arithmetical rules should be borrowed from the objects with which he is most familiar, and proposed in low numbers. The great advantage of an early application to arithmetic is the exercise which it affords to the thinking faculty.

by the common name of *factors*, the *multiplicand* is that number which we want to add repeatedly to itself; and the *multiplier* expresses the number of times that the former is to be repeated in that addition: the sum required is called the *product*.

Thus, by the product of 7 multiplied by 4 we are really to understand the sum of 4 sevens, or $7+7+7+7=28$.

21. The product of any two numbers is the same, whichever of them be made the multiplier.

For instance, if we multiply 8 by 5, we shall have the same product as if we multiply 5 by 8. It is by no means self-evident, that the sum of 5 eights must be the same with the sum of 8 fives, or that $8+8+8+8+8=5+5+5+5+5+5+5+5$; which is the meaning of the proposition. However, it admits of a very easy proof from the following illustration. Suppose 5 rows of 8 counters, regularly disposed under each other. Whatever way we count them, the total amount of the number must be the same. But counting them one way, we have 5 times eight; and counting them another way, it is plain that we have 8 times five counters. It is obvious that a similar proof would be applicable to any higher numbers.

The sign of multiplication is $\times$, interposed between the factors; and is to be carefully distinguished from the sign of addition $+$. Thus, 12×8, or 8×12, denotes the product of 8 and 12.

22. The product of any two numbers is equal to the sum of all the products obtained by multiplying all the parts, into which either is divided, by the other, or by each of the parts into which the other is divided.

Thus, if we suppose 8 to be divided into the parts 4, 3, and 1; the product of 5 times 8 will be equal to the sum of the three products, 5 times 4, 5 times 3, and 5 times 1. And if we suppose the multiplier 5 also divided into the two parts 3 and 2, the product of 5 times 8 will be equal to the six products obtained by multiplying each of the three component parts of the multiplicand by each of the two component parts of the multiplier. The truth of this will appear very plain, by employing the same illustration that was adduced in § 21. In the 5 rows of 8 counters, aptly representing 5 times 8, let us suppose, first, two lines drawn

downwards, dividing each row of eight counters into the three parts 4, 3, and 1. It is then plain that the whole set of 5 times 8 counters is divided into three sets of 5 times 4, 5 times 3, and 5 times 1. Then supposing a line drawn across and dividing each row of 5 counters into 3 and 2, it is plain that each of the three former sets will be divided into two, 3 times 4 and twice 4, 3 times 3 and twice 3, 3 times 1 and twice 1: so that the sum of these six sets is equal to the one set of 5 times 8 counters. This proof is exhibited to the eye in the subjoined scheme.

0000	000	0
0000	000	0
0000	000	0
——	——	—
0000	000	0
0000	000	0

And it is plainly applicable to any other numbers, divided into any parts whatsoever. Thus, if we suppose 17 broken into the four parts 6, 5, 4, and 2; and 9 broken into the three parts 4, 3, and 2; the product of 9 times 17 must be equal to the sum of each of the twelve products obtained by multiplying each of the four parts of the multiplicand by each of the three parts of the multiplier: that is, $17 \times 9 = (24+20+16+8)+(18+15+12+6)+(12+10+8+4)$, or $153=68+51+34$. With the principle brought forward in this section the student cannot be too familiar; as it is the foundation of common multiplication and algebraic, as well as fruitful in the most important inferences.

23. If the multiplier be the product of any two known numbers, we may employ a successive multiplication by the factors of which the multiplier is the product.

Thus, if we want to multiply any number by 54, we may multiply it by 9, and that product by 6; for, 6 times 9 being 54, when we first find a number that is 9 times the multiplicand, and then multiply that number by 6, the product must be 6 times 9 times, or 54 times the multiplicand.

24. The product of any number multiplied by 10, 100, 1000, &c. is obtained at once by annexing one, two, three, &c. ciphers to the multiplicand on the right hand.

Thus, the product of 327 multiplied by 1000 is 327000;

for each digit of the multiplicand is increased in value 1000 times. And combining the principle of the last section, it is plain that if the multiplier be 20, 300, 4000, &c. we may obtain the product by annexing one, two, three, &c. ciphers, and then multiplying by 2, 3, 4, &c. Thus, $4296 \times 700 = 429600 \times 7$.

From the principle stated in § 23, it is manifest that we can find the product of any two numbers: for however great the factors, they may be broken into parts not exceeding 12. The product of all such parts are furnished by the *multiplication table*, which should be committed to memory.

MULTIPLICATION TABLE.*

twice	3 times	4 times	5 times	6 times	7 times
1=2	1=3	1=4	1=5	1=6	1=7
2=4	2=6	2=8	2=10	2=12	2=14
3=6	3=9	3=12	3=15	3=18	3=21
4=8	4=12	4=16	4=20	4=24	4=28
5=10	5=15	5=20	5=25	5=30	5=35
6=12	6=18	6=24	6=30	6=36	6=42
7=14	7=21	7=28	7=35	7=42	7=49
8=16	8=24	8=32	8=40	8=48	8=56
9=18	9=27	9=36	9=45	9=54	9=63
10=20	10=30	10=40	10=50	10=60	10=70
11=22	11=33	11=44	11=55	11=66	11=77
12=24	12=36	12=48	12=60	12=72	12=84

8 times	9 times	10 times	11 times	12 times
1=8	1=9	1=10	1=11	1=12
2=16	2=18	2=20	2=22	2=24
3=24	3=27	3=30	3=33	3=36
4=32	4=36	4=40	4=44	4=48
5=40	5=45	5=50	5=55	5=60
6=48	6=54	6=60	6=66	6=72
7=56	7=63	7=70	7=77	7=84
8=64	8=72	8=80	8=88	8=96
9=72	9=81	9=90	9=99	9=108
10=80	10=90	10=100	10=110	10=120
11=88	11=99	11=110	11=121	11=132
12=96	12=108	12=120	12=132	12=144

* Though the part of the multiplication table given in the text is quite enough for the pupil to commit to memory at first; yet, at [illegible] he has

25. But when the factors, either or both of them, exceed 12, the most convenient parts into which we can conceive them broken are those indicated by the digits.

Thus, if we want to find the product of 537 multiplied by 9, we conceive the multiplicand divided into the parts 7, 30, and 500; and the product is equal to the sum of the three products 9 times 7, 9 times 30, and 9 times 500; each of which the multiplication table furnishes. For 30 being 3 tens, 9 times 30 must be 27 tens, or 270; and 9 times 5 hundreds must be 45 hundreds, or 4500. The product sought therefore must be the sum of the three products 63+270+4500, that is 4833.

This addition of the successive products, by proceeding from right to left in taking the parts of the multiplicand, we are able to perform mentally, without writing the whole of each product separately.

Now, if we want to find the product of 537 multiplied by 69, we suppose the multiplier also divided into the two parts 9 and 60; and having found the product of 9 times the multiplicand, we proceed to find the product of 60 times the multiplicand by § 24, writing the latter product, 32220, under the former, preparatory to the addition of the products. It is plain, therefore, that the cipher annexed to the multiplicand for multiplying by 10 must stand in the column of units, and be preceded by the digits expressing the product of 6 times the multiplicand. But as that cipher will make no change in the subsequent addition, it is commonly omitted; taking care, however, to place the next digit in the column of tens. In like manner, if the multiplier was 469, after having found the two former products, we proceed to multiply by 400. Supposing two ciphers annexed to the multiplicand, and then multiplying by 4, and writing this product, 214800, under the second, preparatory to the addition of the three products,—thus,

made some proficiency in arithmetic, it may be found of advantage to require him to commit 13 times, 14 times, 15 times, 16 times, 17 times, 18 times, and 19 times the eight figures 2, 3, 4, 5, 6, 7, 8, and 9; which the teacher can readily supply.

537	*multiplicand,*
469	*multiplier,*
4833	*product by* 9,
32220	*product by* 60,
214800	*product by* 400,
251853	*whole product by* 469.

The young arithmetician should for some time be made to write the ciphers standing on the right hand of the successive products, that he may be convinced of the reason of the rule which directs us to recede one figure towards the left hand in writing the several products obtained in multiplying by the successive digits of the multiplier.

The child should be taught to prove the accuracy of his work in multiplication by addition, so far as to convince him that the one is but an abridged method of performing the other; and by resolving either or both factors into other parts than those indicated by the digits.

We have noticed the reason of proceeding from the right hand to the left of the multiplicand. But it is generally indifferent in what order we take the digits of the multiplier: and it will sometimes afford a convenient abbreviation to depart from the usual order. Thus, if our multiplier be 945, instead of obtaining the product sought by three distinct products, two will be sufficient by commencing from the left hand of the multiplier; since having found the product of 9 times the multiplicand, 5 times that product will give us the product of 45 times the multiplicand. But when this method is employed, it is plain that the ciphers, which are usually omitted, ought to be expressed.

We have seen, Art. 24, the facility with which multiplication proceeds, when the multiplier consists of a significant figure followed by any number of ciphers. Now, if the multiplier be within twelve of any such number, we may avail ourselves of a convenient abbreviation. For instance, if the multiplier be 4989, we may observe by inspection that this number is within 11 of 5000. If, then, we take 5000 times the multiplicand, and subtract from that product 11 times the multiplicand, the remainder must be 4989 times the multiplicand; or must be the product sought.

Such abridged methods of operation are useful for exer-

cising youthful ingenuity; but ought not to be prematurely introduced. Rational theory, going hand in hand with practice, will soon make the student expert in discerning various advantages which may be taken. For example, if we have to multiply 123456789 by 107988, the multiplier being within 12 of 108000, and 9 times 12 being 108, we may first find 12 times the multiplicand, and subtracting that product from 9000 times that product will give the remainder 13331851730532, for the product sought.

But in general, it is useless to occupy the learner's time in arithmetical operations on numbers so high as scarcely ever occur in real practice. A much more advantageous exercise is to engage him in operating on low numbers *mentally*, without committing them to paper. For instance, to find the product of 25 times 36. This is calculated to form a habit of fixed attention, and to strengthen the mental powers.

Rule for Multiplication of Whole Numbers.

26. Place the multiplier below the multiplicand, with units under units, tens under tens, &c. If the multiplier does not exceed 12, multiply, by means of the multiplication table, each figure of the multiplicand by it, beginning with the units, and setting down and carrying as in addition: the result will be the product required.

But if the multiplier be greater than 12, find the products of the multiplicand by the several figures of the multiplier successively; setting the right hand figure of each product under that figure of the multiplier which produces it: the sum of all these products will be the total product required.*

If either the multiplicand, or the multiplier, or both, end in ciphers, the significant figures may be arranged and multiplied according to the rule, and as many ciphers annexed to the product as are found at the end of both factors. Ciphers in any other part of the multiplier are to be neglected.

* The reason of this rule is evident from the articles in the preceding part of this section.

Examples in Multiplication of Whole Numbers.

Ex. 1. Multiply 324 by 8.

Set the numbers as in the margin, and proceed thus: 8 times 4 are 32, set down 2 and carry 3; 8 times 2 are 16 and 3 are 19, set down 9 and carry 1; 8 times 3 are 24 and 1 are 25, set down 25; and the product is two thousand five hundred and ninety-two, the same that would have been obtained by adding together eight lines, each 324: and thus it would be well to cause the learner to work this and a few similar questions, to make him sensible of the identity of multiplication and addition.

Multiplicand	324
Multiplier	8
Product	2592

Ex. 2. What is the product of 29 and 365?

Set the numbers as in the margin, multiply by 9, and set 5, the first figure of the product, under the 9; then multiply by 2, and set 0, the first figure, under the 2. The sum of these partial products will give ten thousand five hundred and twenty-five, the entire product required.

Multiplicand	365
Multiplier	29
	3285
	730
Product	10585

Ex. 3. Multiply 36407 by 40206.

Here, the first figure of the first partial product is set below the figure 6 in the multiplier; the first of the second partial product below 2; and the first of the third below 4; the ciphers in the multiplier being neglected.

Multiplicand	36407
Multiplier	40206
	218442
	72814
	145628
Product	1463779842

Ex. 4. Multiply 320 by 2400.

The numbers being arranged as in the margin, multiply as in the last example 32 by 24: to 768, the product, add three ciphers, and the entire product is seven hundred and sixty-eight thousand.

Multiplicand	320
Multiplier	2400
	128
	64
Product	768000

Method of Proof.

1. Let the multiplier be taken as multiplicand, and the multiplicand as multiplier; and if the product thus obtained agree with the product found before, the process is right.

Thus, when 64 is multiplied by 45, the product is found to be 2880; and when 45 is multiplied by 64, the same result is obtained.

64	45
45	64
320	180
256	270
2880	2880

2. Add together all the digits of the multiplicand, except 9, which is to be neglected when it occurs; reject also 9 from the successive sums of the digits, when those sums exceed 9, and reserve the final excess. Proceed in like manner with the multiplier and the product. Then multiply together the excesses found in the multiplicand and the multiplier, and, in the same manner as before, find the excess in this product. If this be the same as the excess in the product of the given factors, the work is, in most instances, correct: *if it differ, the work must be wrong.**

* From the great brevity and facility of this mode of proof, it is very convenient in practice. In some cases, indeed, operations which are incorrect may appear by this method of proof to be correct; an occurrence, however, which is extremely rare, unless it be either purposely effected, or arise from the misplacing of figures; and hence, for the purposes of the experienced arithmetician, it is perhaps preferable to any other method.

The principle on which this process depends, is, that *if any number and the sum of its digits be both divided by 9, the remainders, in both cases, are the same.* Thus, $1000=999+1$, where the remainder must be 1, since 999 is evidently divisible by 9, without remainder. Hence, since $3000=1000\times3=999\times3+3$, the remainder here will evidently be 3, since 999×3 is divisible by 9, without remainder.

In like manner, it might be shown that in dividing by 9, the remainder in 5000 would be 5, in 400, 4, in 2000, 2, &c.; being, in all cases, the same as the significant figure contained in the number. Hence, if the number 487 be proposed, it is equivalent to $400+80+7$; which parts, if divided successively by 9, would leave, by the above principle, the remainders 4, 8, 7; and therefore the remainder of the entire number will evidently be the same as the sum of these, diminished by the rejection of 9 as often as possible.

Now, suppose it were required to multiply 112 by 48, $112=12\times9+4$, and $48=5\times9+3$: the product will therefore be equal to $12\times9+4$

In the annexed example, 6 and 8 are 14, which exceeds 9, by 5; 5 and 5 are 10, which exceeds 9, by 1; 1 and 7 are 8, which excess is set opposite to the multiplicand. In the multiplier, in like manner, 8 and 7 are 15; which exceeds 9, by 6, which excess is set opposite to the multiplier. In the product, 6 and 1 are 7 and 5 are 12, which exceeds 9, by 3; 3 and 3 are 6 and 1 are 7 and 5 are 12, which gives an excess of 3, to be set opposite to the product. Then, the product of the two excesses 6 and 8 is 48, the sum of the digits in which is 12, which gives an excess of 3, the same as the excess of the product; and hence we judge the work to be correct.

```
    68597 .... 8
      897 .... 6
   ------
   480179
  617373
 548776
 --------
 61531509 .... 3
 --------
```

Exercises in Multiplication of Whole Numbers.

Ex. 1. 365×2	Ex. 10. 1827×12
2. 144×3	11. 7603×10
3. 123×4	12. 3456×20
4. 345×5	13. 70306×30
5. 456×6	14. 13716×40
6. 789×7	15. 37067×500
7. 2345×8	16. 79634×6000
8. 5678×9	17. 10030×700
9. 3067×11	18. 90320×9000

Examples.		*Answers.*
19. 148×53	- - - - - - -	=7844
20. 958×34	- - - - - - -	=32572
21. 7198×216	- - - - - - -	=1554768
22. 31416×175	- - - - - -	=5497800
23. 40930×779	- - - - - -	=31884470
24. 12345×686	- - - - - -	=8468670
25. 46481×936	- - - - - -	=43506216

multiplied by 5×9+3; which, by multiplying the several terms, becomes 12×9×5×9+5×9×4+12×9×3+4×3. In this product, the first three terms are evidently divisible by 9, without remainder: consequently, the remainder obtained by dividing the whole product by 9, will be the same as the remainder obtained by dividing 4×3, the product of the excesses of the factors, by 9; and thus the reason of this method of proof is evident. The same property belongs to the digit 3, but it is more convenient in practice to use 9.

Examples.		*Answers.*
26.	8320900×1328 - - - -	=11050155200
27.	15607×3094 - - - - - -	=48288058
28.	16734×708 - - - - - -	=11847672
29.	204053×1617000 - - -	=329953701000
30.	9507340×7071 - - - -	=67226401140
31.	39948123×6007 - - - -	=239968374861
32.	57902468×5008 - - - -	=289975559744
33.	92948789×7043 - - - -	=654638320927
34.	144×144×144 - - - -	=2985984
35.	3851×3851×3851. - - -	=57111104051
36.	79094451×764095 - - -	=60435674536845

37. Multiply fifty-six millions seven thousand eight hundred and fifty-four by eight millions six hundred thousand nine hundred and seventy-six. Ans. 45142876960655504.

38. Multiply eighty millions seven thousand six hundred by eight millions seven hundred and sixty.

Ans. 640121605776000.

39. How many yards of linen are in 759 pieces, each containing 25 yards? Ans. 18975.

40. Sound is known to move about 1130 feet per second; how many feet will it move in 69 seconds? Ans. 77970.

41. During the 50 years between 1700 and 1750, the quantity of linen exported from Ireland, each year, at an average, was four millions of yards; during the next six years, 11796361 yards in each; during the next seven, 17776862 yards per year; and during the succeeding seven years, ending 1770, the average quantity was 20252239 yards annually. Required the whole quantity exported from 1700 till 1770. Ans. 638565684.

42. A boy can point sixteen thousand pins in an hour: how many will he do in six days, supposing he works eleven clear hours in a day? Ans. 1056000.

43. How many miles will a person walk in fifty-five years, supposing he travels, one day with another, six miles, and there are 365 days in a year? Ans. 120450.

44. A man spends 99 cents a day: how much does he spend in 49 years, each consisting of 365 days?

Ans. 1770615.

45. Supposing that one acre of land produces 30 bushels of wheat; how many bushels will thirty thousand three hundred and sixty-five produce? Ans. 910950.

Questions.

What is multiplication?
What is the multiplicand?
What is the multiplier?
What is the product?
Is the product of any two numbers the same, whichever of them be made the multiplier?
How is the product of any number multiplied by 10, 100, 1000, &c. obtained?
Repeat the rule for performing the operation of multiplication.
If the multiplier be the product of any two known factors, how is the operation performed?

DIVISION OF WHOLE NUMBERS.

27. *Division*, in the primary view of it, is but an abridged method of subtraction. Here we inquire how often one number, called the *divisor*, may be subtracted from another number, called the *dividend*. The *quotient* expresses the number of times that the divisor may be subtracted from the dividend, or is contained in it.

Thus, when we divide 96 by 12, the quotient is 8: for we may subtract the divisor 12 from the dividend 96 just 8 times. This might be ascertained by performing the successive subtractions, and reckoning the number of them: but is at once discovered by the multiplication table, which informs us that 96 is equal 8 times 12, and therefore contains 12 in it exactly 8 times. If we divide 103 by 12, it is plain that after subtracting 12 from 103 eight times, there will remain 7; so that the quotient is still 8, but with 7 for a remainder.

28. When one number is contained in another a certain number of times exactly, without leaving any remainder, the former number is said to *measure* the latter.

Thus, 12 measures 96, but does not measure 103. The numbers 8 and 12 measure 24; 8 being contained in it exactly 3 times, and 12 exactly twice.

We often express division by writing the dividend above the divisor, with a line interposed between them. Thus, $\frac{84}{7}$ expresses the division of 84 by 7: and the following symbols, $\frac{84}{7}=12$, express therefore that the quotient of 84 divided by 7 is equal to 12. The symbol $\div$ also is sometimes employed to express division, the dividend standing on the left hand of it, and the divisor on the right. Thus, $42\div6$ is another way of expressing the division of 42 by 6, as well as $\frac{42}{6}$.

We shall have such frequent occasion for the signs $+$, $-$, $\times$, $\div$, and the terms *plus* and *minus*, that the young arithmetician cannot too soon become familiar with them. A little patient explanation and illustration will soon make a child as familiar with them as with the Arabic characters; and it is ridiculous to think how many have been deterred from attempting the study of algebra, by the mere formidable appearance of its outworks, a number of strange symbols and terms which they do not understand. But every thing the most simple is obscure till it is understood; and every term is alike unintelligible, till its meaning is explained.

29. If any quotient be made the divisor of the same dividend, the former divisor will be the new quotient, and the same remainder (if any) as before.

Thus, dividing 103 by 12, the quotient is 8, with the remainder 7. Now, if we divide 103 by 8, the quotient must be 12, leaving the same remainder. For the first division shows that the divisor contains 12 eight times, and 7 over; 8 times 12 and 12 times 8 being equal. And thus also it is manifest that if any product be divided by either of the factors, the quotient must be the other factor; and that any dividend may be considered as the product of the divisor and quotient, with the remainder (if any) added.

In the view of division which has hitherto been proposed, the divisor must be conceived not greater than the dividend; else it would be absurd to inquire how often it is contained in the dividend. But there is another view of division, closely connected with the former, in which we may easily conceive the division of a smaller number by a greater. When we are called to divide 96, for instance, by 12, we may consider ourselves called to divide 96 into twelve equal parts, and to ascertain the amount of each. The quotient, found as before, is a number of that amount, or the twelfth

part of 96. For, since 96 contains in it just 8 twelves, it must contain just 12 eights; and therefore the quotient 8 is the twelfth part of 96. And thus universally, the quotient may be considered as that part, or sub-multiple, of the dividend which is denominated by the divisor, as the divisor may be considered that part, or sub-multiple, of the dividend which is denominated by the quotient. (Hitherto, the divisor is supposed to measure the dividend.) Thus, dividing 64 by 4, the quotient is 16; for, subtracting 16 fours from 64, there is no remainder; therefore, 4 is the sixteenth part of 64; and 16 is the fourth part of 64. Now, though it would be absurd to inquire how often 12 may be subtracted from 7, and therefore any division of 7 by 12 is inconceivable, according to that view; yet it is not absurd to inquire what is the twelfth of 7, or of dividing 7 by 12 according to the latter view.

For instance, we might have occasion to divide 7 shillings among 12 persons equally, or into 12 equal shares; and then it is plain that each person must get the twelfth part of 7 shillings. The quotient or twelfth part of 7, as has been already observed, may be represented by the notation $\frac{7}{12}$; and the child ought to be familiarized to this notation, previous to his entrance on the doctrine of fractions.

Let us now revert to the example of division at the close of § 27, the division of 103 by 12. The quotient being 8, but leaving a remainder of 7. Therefore, 8 is not exactly the twelfth part of 103: for if we were dividing 103 dollars equally among 12 persons, after giving each of them 8 dollars, there would be 7 dollars; which 7 dollars we would proceed to divide equally among them; that is, we should give each of them the twelfth part of 7 dollars in addition to the 8 dollars he had received, in order to make the division accurate: therefore, the twelfth part of 103 is exactly $8+\frac{7}{12}$; or 8 and the twelfth part of 7. And so, whenever there is a remainder on a division, the student should be taught to correct the quotient by annexing to it that remainder divided by the divisor.

As to the practical method of performing division, the grounds of it are obvious from § 27. Let us first suppose that the divisor does not exceed 12: for instance, let it be required to divide 5112 by 8. We immediately know from the multiplication table, that 8 may be subtracted 600 times from the dividend, but not 700 times; since 600 times

8 (or 8 times 600) is 4800, but 8 times 700 is 5600, a number greater than the dividend. Subtracting, therefore, 4800 from 5112, there remains 312; and this one subtraction saves the trouble of 600 distinct subtractions of 8 from the dividend. We proceed now to the remainder 312, and consider, from the multiplication table, what is the greatest number of times that 8 is contained in it, or may be subtracted from it; and we immediately know, as before, that 8 is contained 30 times in 312, but not 40 times; 30 times 8 being 240, but 40 times 8 being 320, a number greater than 312. Subtracting, therefore, 30 times 8, or 240, from 312, there remains 72: in which remainder we see that 8 is contained just 9 times. Thus we have ascertained that from 5112, 8 may be subtracted 600 times, 30 times, and 9 times, or in all 639 times; which number is therefore the quotient, and the eighth part of 5112. If the dividend was 5119, it is plain that the quotient would be 639, with the remainder 7; and therefore that the eighth part of 5119 is $639+\frac{7}{8}$. In practice, as we proceed in the operation, the successive multiplications and divisions are performed mentally; attending only to that part of the dividend which ascertains the successive digits of the quotient, and writing only those digits. But the learner ought to be exercised for some time in performing the operation at large, as it has been described; that he may be grounded in the rational principles upon which the practical contraction rests.

The practical operation of division, when the divisor does not exceed 12, is usually set down in the following manner: for instance, the division of 5112 by 8 may be expressed thus,

```
divisor  8)5112  dividend
           ----
            639  quotient.
```

Or thus,

```
                 dividend
       divisor  8)5112(600
      600×8=4800   30
            ----    9
             312  ---
       30×8= 240  639  quotient.
             ---
              72
        9×8=  72
              --
```

```
Or thus,            dividend
          divisor   8)5112(639   quotient.
                      48
                      ---
                       31
                       24
                       ---
                        72
                        72
                        ---
```

Here, it is manifest that the principles of those three different methods are the same, but according to the first method, there is more of the operation to be performed mentally.

Let us now suppose that the divisor exceeds 12; for instance, that we have to divide 27783 by 49. We may at once conclude that the quotient must be less than 700, as 700 times 40 (or 28000) would exceed the dividend, and therefore much more 700 times 49. But the dividend does not contain the divisor even 600 times; for though 600 times 40 (or 24000) is less than the dividend, yet 600 times 49 will be found greater than the dividend. (Nothing but practice can make the student quick in perceiving this; and he may for a time have the trouble of trying numbers in the quotient, which he will find to be too great.) Subtracting, therefore, 500 times the divisor, or 24500, from the dividend, there remains 3283; from which we subtract 60 times the divisor, or 2940. In the remainder 343, we find that the divisor is contained just 7 times; so that the entire quotient is 567.

```
                  dividend
       divisor  49)27783 ( 500
                   24500 {  60
                   ----- (   7
                    3283   ---
                    2940   567   quotient.
                    ----
                     343
                     343
                     ---
```

In such instances of what is called *long* division, it is necessary to write the successive remainders. But after the student has been well grounded in the principles of the ope-

ration, it will be expedient that he should perform the subtractions without writing the successive products; subtracting the several digits composing them as he proceeds with the multiplication.

Thus, it appears that we are enabled by the multiplication table to determine the successive digits of the quotient from the left hand.*

Rule for Division of Whole Numbers.

30. Place the divisor to the left of the dividend, with a line between them, and leave a space to the right of the dividend for containing the quotient.

Find, by the multiplication table, how often the first figure of the divisor is contained in the first figure, or the first two figures of the dividend, and set the figure denoting the number of times in the quotient.

Multiply the divisor by the figure thus found, set the product below the leading figures of the dividend, and subtract it from them. To the remainder annex the next figure of the dividend, divide the result as before, and thus proceed till the operation is finished.

If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value; but if any remainder be greater than

* Although the order of proceeding which is above described be the most convenient, yet the young arithmetician should be practised in resolving the dividend differently, and proceeding on similar principles, but in another order: for instance, in dividing 27783 by 49, instead of first dividing the component part 2700 by 49, and then incorporating the remainder 2500 with the other component part 783, as above, the same result may be obtained by commencing with the latter component part of the dividend 783. Dividing it by 49, the quotient is 15, with the remainder 48; adding this remainder to the other part of the dividend 2700, we may proceed in like manner to ascertain how many times 49 is contained in their sum, by commencing with the component part 7048: the quotient will be 143, with the remainder 41. And adding the remainder to the 20000 which has not yet been divided, 49 will be found to be contained in their sum 20041 just 409 times. Now the sum of the three quotients, 15+143+409, is 567, as before. And thus the student may be taught to prove the accuracy of his work in division, not only by multiplying the divisor and quotient, but also by resolving the dividend into any two or more parts, dividing each of them by the given divisor, and adding the quotients.

the divisor, or equal to it, the last figure in the quotient must be changed for a greater. If any of the successive dividends be less than the divisor, a cipher must be put in the quotient, and another figure, if any remain, brought down from the given dividend.

When the divisor is below 13, the several multiplications and subtractions may be performed mentally, as has been already observed, and the quotient set under the dividend: this operation is usually called *short divison.*

Examples in Division of Whole Numbers.

Ex. 1. Divide 136 by 8.

Here we say, 8 in 13 once, and 5 remain; 8 in 56 seven times, and nothing remains. The quotient therefore is 17; and by multiplying it by the divisor, 8, we obtain 136, the dividend, which proves the operation.

```
8)136
  ———
   17 Quotient.
    8
  ———
  136 Proof.
```

Ex. 2. Divide 15967 by 57.

Let the divisor, 57, be set before the dividend, 15967, as in the margin, and proceed thus: How often is 5 contained in 15?—twice:* place 2 in the quotient, multiply the divisor by it, and set the product below 159, the leading part of the dividend. This being subtracted from 159, the remainder is 45, to which 6, the next figure of the dividend, is annexed. Again, how often 5 in 45?—8 times: place 8 in the quotient, proceed as before, and there is no remainder. Then 7, the remaining figure of the divi-

```
57)15967(280 7/57
   114··
   ———
    456
    456
    ———
      7
```

* It would seem here at first sight, that 3, and not 2, should be put in the quotient. Were the divisor multiplied by 3, however, the product would be greater than 159. The learner often finds difficulty in this way, in discovering what figure he should place in the quotient. In this example, 5 is contained in 15 twice, with the remainder 5, which with 9, the next figure in the dividend, makes 59. In this, 7, the second figure of the divisor, is contained 8 times, which being more than 2, the number of times 5 was found in 15, we are certain that the divisor is contained twice. Had 5 been taken 3 times in 15, however, we should have had no remainder to 9; and 7, the second figure, is not contained 3 times in 9, and consequently the whole divisor is not contained 3 times in the leading figures of the dividend. This method may perhaps be employed with some advantage when the divisor is large; practice, however, will soon render it unnecessary.

When the second figure of the divisor is above 5, in trying for the

dend, containing 57 no times, a cipher is placed in the quotient, and the remainder is written in the quotient over the divisor 57: the quotient, therefore, is $280\frac{7}{57}$.

Methods of Proof.

1. Find the product of the divisor and quotient; and add to it the remainder; if the sum be equal to the dividend, the work is correct.

2. Subtract the remainder from the dividend, and divide the result by the quotient; if the quotient thus found be the same as the original divisor, the work is right.

3. *Cast out the nines* from the divisor, dividend, quotient, and remainder; then to the product of the excesses of the divisor and quotient, add the excess of the remainder, and cast the nines out of the sum; if this excess be equal to the excess of the dividend, the work is generally correct.

The reason of the first and second methods of proof has been already given in the preceding articles of this section; and the reason of the third is also evident from the note, page 32.

31. If the given divisor be the product of any two or more known factors, the quotient may often be more expeditiously obtained from successive division by those factors.

Thus, in dividing 27783 by 49; since 49 is 7 times 7, if we divide 27783 by 7, and again divide the quotient, 3969,

figure to be placed in the quotient, the first may be increased by unity: thus, in the example before us, we might have said, how often 6 in 15, &c.

Another method which may be also used by the learner with some advantage, is, to multiply the divisor by the nine digits, and set down the respective products as in the margin; then it can be readily seen, which of these products is next less than the leading figures of the dividend; place the corresponding integer in the quotient, set the product below the leading figures of the dividend, subtract it from them, bring down the next figure of the dividend, and proceed as before.

```
49)27783(567 Quotient.
   245··
   ---
    328
    294
    ---
     343
     343
     ---
```

$49\times1=$	49
$49\times2=$	98
$49\times3=$	147
$49\times4=$	196
$49\times5=$	245
$49\times6=$	294
$49\times7=$	343
$49\times8=$	392
$49\times9=$	441

by 7, we shall have the result. 567. The reason of this is plain; because 27783 is 7 times the first quotient; and the first quotient, 3969, is 7 times the second quotient. Therefore the given dividend is 49 times the second quotient, or 567 is the 49th part of the given dividend.

$$\begin{array}{r} 7)27783 \\ \hline 7)3969 \\ \hline 567 \end{array}$$

The number 7, being seven times less than 49, must be contained in the dividend 7 times oftener. But 7 is contained in 27783 just 3969 times. Therefore 49 must be contained in it the 7th part of 3969 times, or the quotient sought is the 7th part of 3969.

But when this method is employed, we must carefully attend to the management of the remainders. Thus, dividing 5689 by 42, the quotient is 135, with the remainder, 19; and we employ a successive division by 7 and 6, the first quotient will be 812, with the remainder of 5, and dividing that quotient by six, we shall get the quotient 135, with the remainder of 2. But this 2 is to be considered as 2 sevens, or 14; which, added to the former remainder, gives 19 for the true remainder, as before. The reason of this will be plain from considering that by the first division we find that the dividend contains in it 812 sevens: so that any remainder on dividing that 812, must be regarded as of the denomination *sevens.**

32. Any number is divided by 10, 100, 1000, &c. by cutting off as many digits from the right hand of the dividend, as there are ciphers in the divisor. The digits thus cut off express the remainder, and the remaining digits of the dividend the quotient.

Thus, dividing 234567 by 1000, the quotient is 234, with the remainder 567. This is manifest, since the dividend is equal to 1000 times 234, with 567 added to the product.

* This may be made quite clear to the youngest student by supposing that we wanted to divide 53 dollars by twelve; that is, to find how many sets of 12 dollars are contained in 53 dollars. Dividing 53 by 4, we find that it contains 13 sets of 4 dollars each, and one over. Every three of this quotient will make a parcel of 12 dollars; and now to find their number, dividing 13 by 3, the quotient is 4, (four parcels of 12 dollars) and one over. But this one is plainly one set of 4 dollars: which added to the former one dollar, gives 5 for the remainder, and 4 for the quotient. Hence appears the reason of the rule which directs us *to multiply the remainder on the second division by the first divisor, and add the product to the remainder on the first division.*

Hence, it is plain that if the divisor consist of any significant figures, followed by any number of ciphers, we may employ the method of division described in the last article. Thus, if we want to divide 234567 by 7000, we may divide first by 1000 and then by 7 ; and the quotient will be 33, with the remainder 3567. For when we divide 234 by 7, the remainder of 3 is in fact 3000, and is to be added to the first remainder, 567. And we shall have the same result, (though not so expeditiously) if we first divide by 7, and then by 1000.

When the pupil has had some practice in the methods already explained, he may be taught (as has been already observed) to omit writing the products, which will at least save much room in his operations. This method will be understood from the following example.

Ex. 3. Divide 59122 by 82.*

```
82)59122(721
     ..
    ----
    172
    ----
     82
     --
     ..
```

Here, the first figure put in the quotient is 7; then we say, 7 times 2 are 14, 14 from 21 and 7 remain; 7 times 8 are 56 and 2 (carried) are 58 from 59 and 1 remains. We then bring down 2, and place 2 in the quotient; then, twice 2 are 4, 4 from 12 and 8 remain, twice 8 are 16 and 1 (carried) are 17, 17 from 17 and nothing remains. Bring down 2, and place 1 in the quotient; then 82 from 82 and nothing remains.

Although the principle on which the operations in division depends has been already explained in § 29 ; still the demonstration alluded to may be more clearly understood by the following example.

Ex. 4. Divide 8560 by 36.

* The divisor is placed to the right of the dividend by the French, and the quotient below it, as in the margin. This mode gives the work a more compact and neat appearance, and possesses the advantage of having the figures of the quotient near the divisor, by which means the practical difficulty of multiplying the divisor by a figure placed at a distance from it, is removed. This difficulty every one must have felt, particularly in long operations ; and hence this method might, with much propriety, be employed in preference to that which is employed in this country, as well as in England.

```
59122(82
574..  ---
----   721
 172
 164
 ----
   82
   82
   --
   ..
```

36)8560(200	36)8560(237$\frac{28}{36}$
36×200=7200	72..
36)1360(30	136
36×30=1080	108
36) 280(7	280
36×7= 252	252
237	
28	28

Here, the first part of the quotient is 200, the product of which by 36 is 7200. This taken from the dividend leaves 1360 to be divided by 36. The next part of the quotient is 30; the product of which by 36 is 1080; which still leaves a remainder of 280 to be divided by 36. This gives 7, with the remainder 28. Hence, it appears that 36 is contained 200+30+8 times, or 238 times, with the remainder 28. By comparing this and the common process subjoined, it will be found that the latter is merely an abbreviation of this, the ciphers being omitted in the one and retained in the other.

Exercises in Division of Whole Numbers.

Ex. 1. 371484÷2	Ex. 7. 81034÷8
2. 103465÷3	8. 41098÷9
3. 716564÷4	9. 10340÷10
4. 703025÷5	10. 30007÷11
5. 103453÷6	11. 23456÷12
6. 734567÷7	12. 43078÷3

Examples.	*Answers.*
13. 1567894÷20 - - - - -	=78394$\frac{14}{20}$
14. 7037894÷30 - - - - -	=234596$\frac{14}{30}$
15. 3074560÷40 - - - - -	=76864
16. 1000050÷50 - - - - -	=20001
17. 89030580÷60 - - - - -	=1483843
18. 10030700÷70 - - - - -	=143295$\frac{50}{70}$
19. 40100800÷80 - - - - -	=501260
20. 903745678÷90 - - - - -	=10041618$\frac{58}{90}$
21. 103703070÷100 - - - -	=1037030$\frac{70}{100}$
22. 703456600÷110 - - - -	=6395060

Examples.		*Answers.*
23. $103457040 \div 120$	- - -	$=862142$
24. $673454800 \div 16$	- - -	$=42090925$
25. $51846734 \div 102$	- - -	$=508301\frac{32}{102}$
26. $727346489 \div 408$	- - -	$=1782711\frac{491}{408}$
27. $980263711 \div 809$	- - -	$=1211698\frac{29}{809}$
28. $536819237 \div 907$	- - -	$=591862\frac{403}{907}$
29. $1457924651 \div 1204$	- -	$=1210900\frac{1051}{1204}$
30. $28101418481 \div 1107$	- -	$=25385201\frac{974}{1107}$
31. $1111111111111 \div 854$	- -	$=1301066874\frac{715}{854}$
32. $1000000000000000 \div 111$		$=9009009009009\frac{1}{111}$
33. $1000000000000000 \div 1111$		$=9000900090000\frac{1000}{1111}$
34. $1000000000000000 \div 11111$		$=90000900009\frac{1}{11111}$

35. Divide 74638105 by 37. Ans. $2017246\frac{3}{37}$.

36. Divide 31086901 by 7100. Ans. $4378\frac{3101}{7100}$.

37. Divide 7380964 by 23000. Ans. $320\frac{20964}{23000}$.

38. Divide 2304109 by 5800. Ans. $397\frac{1509}{5800}$.

39. Suppose 96000 men are formed into ranks of three deep, what is the number in each rank? Ans. 32000.

40. The distance from New-York to Saratoga is 201 miles; how many miles each day must an army march in order to arrive at the latter place in 3 days? Ans. 67 miles.

41. The annual income of a gentleman being $38330; how much per day is that equivalent to, there being 365 days in the year? Ans. $104.

42. How many lessons of ninety-five lines each are contained in Virgil's Æneid, the number of lines contained in that poem being nine thousand eight hundred and ninety-two? Ans. $104\frac{12}{95}$.

43. If it be supposed, as in common circumstances is found to be nearly true, that as many persons die in 33 years as are equal to the entire population; it is required to find how many persons die each year, at an average, in the United States, the population being ten millions two hundred and thirty thousand? Ans. 310000.

44. A prize, worth $20000 is to be divided equally among 25 men; what is each man's part? Ans. $800.

45. It is estimated that there are a thousand millions of inhabitants in the known world: if a number of persons equal to the whole population die in 33 years, how many deaths are there in a year?

Ans. 30303030, and 10 of a remainder.

46. The national debt of England, at present, cannot be less than five hundred millions sterling: how long would that be in paying at the rate of four millions and five hundred thousand pounds a year? Ans. 200 years.

47. The national debt of the United States, at present, [1827] is about seventy millions five hundred thousand and five hundred dollars: how much must the debt be reduced every year, so that it may be all paid off in fifty years?

Ans. 1410010.

Questions.

What is division?
What is the divisor?
What is the dividend?
What is the quotient?
Repeat the rule of operation.
If the divisor be the product of two or more known factors, how is the operation of division performed?
If the divisor consist of unity with one or more ciphers, how is the operation performed?

CHAPTER II.

PRACTICAL APPLICATION OF MULTIPLICATION AND DIVISION TO THE REDUCTION OF MONEY, WEIGHTS, MEASURES, &C.

33. REDUCTION is the method of converting quantities from one name or denomination to another of the same value; and it is divided into *Reduction Descending*, and *Reduction Ascending*.

When quantities of a higher denomination are to be brought to a lower, it is called Reduction *descending*, and it is performed by multiplication.

When quantities of a lower denomination are to be brought to a higher, it is called Reduction *ascending*, and it is performed by division.

From the first initiation of the youthful student into multiplication and division, he ought to be led to the practical

use of these operations by familiar questions involving low numbers. For instance, he may be called to find how many apples are wanted in order to give 4 a-piece to 16 persons; or called to divide 96 apples equally among 4 persons. Reduction descending and ascending will furnish a variety of useful examples for exercising the student in multiplication and division.

REDUCTION DESCENDING.

To reduce a quantity from a higher to a lower denomination.

34. *Rule.* Multiply the number which expresses the quantity by the number which shows how many of the lower denomination make *one* of the higher; and if any part of the given quantity be already of the lower denomination, add it to the product.

Federal money, or the money of the United States.

35. The denominations of this money are eagle, dollar, dime, cent, and mill.

10 mills, marked	(*m*)	make	1 cent.
10 cents	(*c*)		1 dime.
10 dimes, or 100 cents	(*d*)		1 dollar.
10 dollars	(*D*) or $		1 eagle. (*E*)

Hence the following particular Rule for reducing federal money from a higher to a lower denomination.

Eagles multiplied by		10	give dollars.
Dollars	×	10	dimes.
Dimes	×	10	cents.
Cents	×	10	mills.
Dollars	×	100	cents.

Examples.

1. In 350 eagles and 6 dollars, how many cents?

```
          E      $
         350     6
          10
        -----
        3500 dollars in 350 eagles.
add        6
        -----
        3506 dollars.
          10
        ------
       35060 dimes.
          10
       -------
Answer 350600 cents.
```

Or thus :

```
        3506 dollars.
         100
       -------
      350600 cents.
```

2. How many mills in 365 dollars, 37 cents, and 3 mills ?

```
      $     cts.    m.
     365    37      3
     100
    ------
    36537   cents.
       10
    ------
   365370
        3
   ------
   365573   mills.
```

Here it is proper to observe, that instead of multiplying the dollars by 100, and multiplying the product thus arising, after adding 37 cents, by 10, we may write down the dollars, cents, and mills, as we do whole numbers ; because this money increases anddecreases according to the decimal notation : this is evident from the above example. But, if the cents be less than 10 or, which amounts to the same thing, if there be no dimes, then 0 must be put in the place of dimes. For instance, if it were required to reduce 47 dollars and 7 cents to cents ; we must put 0 in the dimes', or tens' place, and the cents will be expressed ; thus, 4707 cents. And if there are no cents, two ciphers are used.

It may also be observed, that as one dollar is the *unit* of *federal money*, to which all other denominations of the same money are compared, it is usual to separate the dollars from the dimes, cents, and mills, by a point, called a *separatrix:* for instance, 375 dollars, 29 cents, is written thus :

$ 375·29,

$ cts.

and sometimes thus ; 375·29, although it is not necessary to place the mark (cts.) of cents over the cents, when they are separated from the dollars by a point.

In like manner, if it were required to write dollars, cents, and mills, the cents are usually separated from the mills by a point : for instance, 375 dollars, 29 cents, and 3 mills, is written thus,

$ 375·29·3,

$ cts. m.

and sometimes thus ; 375 · 29 · 3.

When the marks of the several denominations are placed over the quantity, as in the last instance, it is not necessary to separate the dollars, cents, and mills, by points ; and it is usually written thus,

$ cts. m.

375 29 3

It is plain that any sum of this money may be considered as mills, or as cents and mills, by removing the point or points, without the operation of reduction: for instance,

cts. m.

$ 375·29·3 is reduced to 37529·3, or 37529 cents and 3 mills, by removing one point; and again, by removing both points, it is reduced to 375293 mills.

This money is usually read and written in dollars and cents, because accounts in the United States are kept in dollars and cents, the mills being generally neglected, or, if they are considered, parts of a cent are used instead of them ; for instance, 5 mills, or any number less, is written half a cent, or $\frac{1}{2}$ cent ; for any number of mills between 5 and 10 a cent is usually counted ; sometimes one-fourth (marked $\frac{1}{4}$) and three-fourths (marked $\frac{3}{4}$) are reckoned.

The eagles and dimes are seldom mentioned in reading this money, the former being considered as tens of dollars, and the latter as tens of cents.

3. Reduce 37 eagles, 7 dollars, and 7 mills to mills.

Ans. 377007 mills.

4. In 47 dollars, how many cents and mills?
Ans. 4700 cents and 47000 mills.

5. Reduce 37 cents to mills. Ans. 370 mills.

6. Reduce 375 dollars and 3 mills to mills.
Ans. 375003 mills.

7. Reduce 976 dollars and 9 cents to cents.
Ans. 97609 cents.

8. In 36 eagles, 17 dollars, and 7 cents, how many cents?
Ans. 37707 cents.

9. In 760 dollars, 3 cents, and 7 mills, how many mills?
Ans. 760037 mills.

English Money.

35. The denominations of *English money* are *pound, shilling, penny, halfpenny*, and *farthing*.

Table.

		make
4 farthings, (*qr.*, $\frac{1}{4}$) or 2 halfpence, ($\frac{1}{2}$)	=	1 penny, (*d.*)
12 pence	=	1 shilling (*s.*)
20 shillings	=	1 pound (£)*

Hence the following Rule for reducing this money from a higher to a lower denomination.

Pounds	multiplied by	20	give shillings.
Shillings	×	12	give pence.
Pence	×	4	give farthings.
Pence	×	2	give halfpence.

Example 1. In 76 pounds, 13 shillings, and 9 pence, how many pence?

* L. s. d. and q. are the initials of the Latin words libra, solidi, denarii, and quadrantes, signifying pounds, shillings, pence, and farthings.

£	s.	d.
76	13	9

20
———
1520 shillings in £76
13
———
1533 do. in £76 13*s.*
12
———
18396 pence in £76 13*s.*
9
———
Ans. 18405 pence in £76 13*s.* 9*d.*

Here it may be observed that 13 may be added mentally to the product of 76 by 20 according as we perform the operation of multiplication, and only the sum is put down; and so on for the shillings and pence: this will abridge the work, and the operation of the preceding example will stand thus;

£	s.	d.
76	13	9

20
———
1533 *s.* in £76 13*s.*
12
———
Ans. 18405 pence, as before.

Here, in multiplying 76 by 20, a cipher must be in the unit's place; therefore, in adding 13 to the product, 3 must be in the unit's place of the sum; hence we put 3 in the unit's place: again, 6×2 tens gives 12 tens, to which 1 ten must be added, and the sum will be 13 tens. We must, therefore, put 3 in the place of tens, and carry 1 to the place of hundreds, and then proceed as in whole numbers. In like manner, the multiplication of 1533 by 12, and the addition of 9 to the product, is performed mentally: the same method may be used in working the following examples:—

2. In £3974 how many farthings? Ans. 3815040.

3. In £99 how many shillings, pence, and farthings?
Ans. 1980*s.* 23760*d.* and 95040*qrs.*

4. How many pence are there in £19, 19 shillings and 11 pence? Ans. 4799 pence.

5. Reduce £99, 19 shillings and $11\frac{3}{4}$ pence to farthings. Ans. 19199.

6. Reduce 19 shillings and $11\frac{1}{2}$ pence to half-pence. Ans. 9799.

7. Reduce $11\frac{1}{4}$ pence to farthings. Ans. 45.

Troy or Goldsmith's Weight.

36. The denominations of this weight are pound, ounce, pennyweight, and grain.

24 grains (*gr.*)	make	1 pennyweight, *dwt.*
20 pennyweights		1 ounce, *oz.*
12 ounces		1 pound, *lb.*

By this weight, gold, silver, jewels, and precious stones are weighed. It is also used to ascertain the strength of liquors.

Rule.

Pounds, Troy, multiplied by		12 give ounces,
Ounces	×	20 give pennyweights,
Pennyweights	×	24 give grains.

Ex. 1. How many grains of gold are there in a cup weighing 3*lb.* 9*oz.* 6*dwts.* 18*grs.*?

lb.	*oz.*	*dwts.*	*grs.*
3	9	6	18
12			
45			
20			
906			
24			
3632			
1813			
21762 grains.			

Here I multiply the 3 by 12, and add 9 to the product for the number of ounces. I then multiply 45 by 20, and add 6 for the pennyweights: and afterwards the 906 by 24, and take in 18 for grains.

Ex. 2. In 1434*lb.* 6*oz.* 7*dwts.* 19*grs.* how many grains? Ans.

Ex. 3. Reduce 105*lbs.* troy into grains. Ans.

Ex. 4. In 36*lbs.* 10*oz.* 14*dwts.* 18*grs.* how many grains? Ans.

Ex. 5. In 11*oz.* 15*dwts.* 23 grains, how many grains? Ans.

Ex. 6. In 500 spoons, weighing 120*lbs.* and 19*dwts.*, how many grains? Ans.

Avoirdupois or Grocers' Weight.

37. The denominations of avoirdupois weight are ton, hundredweight, quarter, pound, ounce, dram.

Table.

16 drams (*dr.*) make	1 ounce, *oz.**
16 ounces	1 pound, *lb.*
28 pounds	1 quarter, *qr.*
4 quarters, or 112*lb.*	1 hundred weight, *cwt.*
20 hundred weight	1 ton, T.

By this weight almost all coarse and heavy goods are weighed; such as groceries, cheese, butter, &c.; wax, pitch, tallow, and all metals, excepting gold and silver.—Flour, beef, and pork, are also weighed by Avoirdupois weight: a barrel of beef or pork=200*lb.*, a barrel of flour =196*lb.*

* The *Avoirdupois* ounce is *less* than the *Troy* ounce; but the Avoirdupois pound is greater than the pound Troy. 175 Troy ounces are equal to 192 Avoirdupois ounces; but 144*lb.* Avoirdupois are equal to 175*lb.* Troy. Therefore 1*lb.* Avoirdupois is equal to 1*lb.* 2*oz.* 11*dwts.* 16*grs.* Troy. Hence the following Table:

144*lb.* Avoirdupois	=175*lb.* Troy.				
192*oz.*	=172*oz.*				
	lb.	*oz.*	*dwts.*	*grs.*	
1*lb.*	=1	2	11	16	=7000*grs.* Troy
1*oz.*	=0	0	18	5½	=437½
1*dr.*	=0	0	1	3⅓	=27·35
	lb.	*oz.*	*drs.*		
1*lb.* Troy	=0	13	2⅔ nearly, Avoirdupois		
1*oz.*	=0	1	1½		

e difference between the pound Avoirdupois and the pound e former contains 7000 grs., the latter only 5760 *grs.*

The hundredweight above mentioned is usually called the *great* hundred, to distinguish it from hundreds of different magnitudes, which are used in particular places. One of the most general of these in the United States, is the *short* hundred, which contains 100*lb.* A ton of stone is 21 hundreds of 120*lb.*, which is called the *long* hundred.

Rule.

Tons multiplied by	20	give	*cwts.*
Hundred weights	×	4	*qrs.*
Quarters	×	28	*lbs.*
Hundred weights	×	112	*lbs.*
Pounds	×	16	*oz.*
Ounces	×	16	*drams.*

The stone, in the greater number of places, is 14*lbs.*, or one-eighth of the standard hundred. 14*lb.* of wool = 1 stone —2 stones = 1 tod—$6\frac{1}{2}$ tods = 1 wey—2 weys = 1 sack—12 sacks = 1 last,—and a pack of wool = 240*lb.* The stone of 14*lb.* is also the standard at Newmarket Races, England, as well as at the races in the United States. A stone of iron or shot = 14*lb.*

Ex. 1. How many drams are there in 25 tons, 15*cwt.* 3*qrs.* 24*lb.* 12*oz.* 8 *drams?*

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
225	15	3	24	12	8

```
      225
       20
   ------
     4515
        4
   ------
    18063
       28
   ------
   144508
   36128
   ------
   505788
       16
   ------
  8092620
       16
  -------
129481928
```

I multiply by 20, and take in the 15*cwt.*, because 20*cwt.* make 1 ton; then by 4 and take in the 3, because 4 quarters make 1*cwt.*; then by 28 and take in the 24, because 28*lb.* make a quarter; then by 16 and take in the 12, because 16 ounces make a pound; and again by 16 and take in the 8, because 16 drams make an ounce.

Ans. 129481928 drams.

2. In 183 *cwt.* how many ounces? Ans. 327936.
3. In 76*cwt.* 2*qrs.* 13*lb.* how many pounds? Ans. 8581.
4. In 3*qrs.* 14*lb.* 13*oz.* how many ounces? Ans. 1581.
5. How many drams in 35 tons, 17*cwt.* 1*qr.* 23*lb.* 7*oz.* 13*dr.*? Ans. 20571005.
6. Reduce 137 tons to hundreds. Ans. 2740.
7. Reduce 47*cwt.* 3*qrs.* 24*lb.* to pounds. Ans. 5372.
8. Reduce 135*cwt.* 3*qrs.* 11*lb.* to pounds. Ans. 15215.
9. Reduce 13*lb.* 4*oz.* 5*dr.* to drams. Ans. 3397.
10. Reduce 1 ton to ounces. Ans. 35840.
11. Reduce 59 tons, 11*cwt.* 8*lb.* to pounds. Ans. 133400.
12. In 17 stone, 13*lb.* 5*oz.* of wool, how many ounces? Ans. 4021.

Apothecaries' Weight.

38. The denominations of Apothecaries' weight are pound, ounce, dram, scruple, and grain.

Table.

20 grains (*gr.*)	make	1 scruple ℈	= 20*gr.* Troy.
3 scruples		1 dram ʒ	= 60
8 drams		1 ounce ℥	= 480
12 ounces		1 pound *lb.*	= 5760*

By this weight apothecaries mix their medicines, but they buy their drugs by Avoirdupois weight. The pound and ounce made use of by apothecaries, and the pound and ounce Troy weight are the same, but the smaller divisions are different;—see note to article 37.

Rule.

Pounds	multiplied by	12	give	ounces,
Ounces	×	8		drams,
Drams	×	3		scruples,
Scruples	×	20		grains.

* Physicians write their prescriptions according to the following table and characters:

20 *grs.* Troy	=1 scruple	℈j	
60	=1 dram	ʒj	=℈iij
480	=1 ounce	℥j	=ʒviij
5760	=1 pound	℔j	=℥xij

Ex. 1. How many grains are there in 3℔, 3℥, 3ʒ, 1℈, and 12*gr.*?

℔	℥	ʒ	℈	*gr.*
3	3	3	1	12

12
—
39
8
—
315
3
—
946
20
—
18932   Ans. 18932 grains.
—

2. In 139℔, 7℥, 2ʒ, 1℈, and 16*gr.* how many grains? Ans 804159.

3 How many scruples are there in 36℔, 9℥ of Peruvian bark? Ans. 10584.

4. In 1℔, 5℥, 3ʒ, how many drams? Ans. 139.

5. Reduce 3℔ to grains. Ans. 17280.

6. Reduce 5℔, 3℥, 5ʒ, and 17*gr.* to grains. Ans. 30557.

Cloth Measure.

39. The denominations of Cloth Measure are yard, quarter, nail, and inch.

Table.

$2\frac{1}{4}$ inches	make	1	nail, *n.*
4 nails, or 9 inches		1	quarter, *qr.*
4 quarters, or 36 inches		1	yard, *yd.*
5 quarters, or 45 inches		1	English ell, *E. e.*

This is used to measure cloth, tape, &c.

Rule.

Yards multiplied by	4	give	quarters,
Quarters	×	4	nails,
English ells	×	5	quarters,
Nails	×	$2\frac{1}{4}$	inches.

Ex. 1. How many inches in length are there in 156 ells of cambric?

ells.	
156	
5	
780	In multiplying by $2\frac{1}{4}$, I first multiply by 2, then *divide* the multiplicand by 4, which is the same as multiplying by $\frac{1}{4}$; add the sums thus found for the true answer.
4	
3120	
$2\frac{1}{4}$	
6240	
780	
7020	Ans. 7020 inches.

2. In 15*yds.* 2*qrs.* 3 nails, 1 inch, how many inches? Ans. $565\frac{3}{4}$.
3. Reduce 28*yds.* 3*qrs.* 2 nails, to nails. Ans. 462.
4. Reduce 58*yds.* to nails. Ans. 928.
5. Reduce 1241 English ells, 2*qrs.* 3 nails, to nails. Ans. 24831.
6. Reduce 40*yds.* 3*qrs.* to inches. Ans. 1417.

Long Measure.

40. The denominations of Long Measure are league, mile, furlong, rod or pole, yard, foot, inch, line.

Table.

12 lines	make	1 inch, *in.*
12 inches		1 foot, *ft.*
3 feet, or 36 inches		1 yard, *yd.*
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet		1 pole or rod, *p.*
7 yards, Irish Measure		1 pole or perch,
40 poles, or 220 yards		1 furlong, *fur.*
8 furlongs, or 1760 yards		1 mile, *m.*
3 miles, or 5280 yards		1 league, *lea.*

A *fathom*, which is used for measuring the depth of water, is 2 yards, or six feet; a *hand*, (used in measuring the

height of horses,) is 4 inches, a span 9 inches. From this Table, by an easy reduction, it will appear that a mile, either American or Irish, contains 320 poles; and that an American mile contains 1760 yards, or 5280 feet, while an Irish mile, in consequence of the difference in the length of the perch, contains 2240 yards, or 6720 feet. It follows, also, that 11 Irish miles are equal to 14 American miles. The British and American pole and mile are of the same length.

Rule.

Leagues	multiplied by	3 give	miles,
Miles	×	8	furlongs,
Furlongs	×	40	poles,
Poles	×	$5\frac{1}{2}$	yards,
Yards	×	3	feet,
Feet	×	12	inches,
Inches	×	12	lines.

Ex. 1. Reduce 53 American or English miles, 3 furlongs, 12 poles, 4 yards, to yards.

M.	*Fur.*	*P.*	*yds.*
53	3	12	4

8

427 furlongs.
40

17092 poles.
$5\frac{1}{2}$

85460
8546

94010 yards,—Ans.

In this example, because the American pole contains $5\frac{1}{2}$ yards, the poles are multiplied by 5, and half of them taken, and the 4 yards are added in with the two results. It may be observed, that if the last figure be rejected from the product by 5, there will remain half the number of poles; and if the figure rejected be 5, it will be half a yard, or a foot and a half.

Ex. 2. Reduce 5 Irish miles, 3 furlongs, 12 poles, 1 foot, to feet.

M.	*Fur.*	*P.*	*yds.*	*ft.*
5	3	12	0	1

8
—
43 furlongs.
40
—
1732 poles.
7
—
12124 yards.
3
—
36373 feet,—Ans.
—

Here the miles are multiplied by 8 to reduce them to furlongs—the furlongs by 40 to reduce them to perches—the perches by 7 to reduce them to yards—and the yards by 3 to reduce them to feet. The reason is evident from what has been said respecting the reduction of money and weights, and from the table of Long Measure.

Ex. 3. Reduce 5 feet, 4 inches to inches. Ans. 64.

4. Reduce 3 Irish miles, 3 furlongs, to yards. Ans. 7560.

5. Reduce 4 miles, 4 furlongs, 20 poles, to poles. Ans. 1460.

6. In 4 leagues, 6 furlongs, and 20 poles, how many poles? Ans. 4100.

7. Reduce 10 American poles, 2 yards, 1 foot, 7 inches, to inches. Ans. 2071.

8. Reduce 25 American miles and 34 poles to feet. Ans. 132561.

Square Measure, or the Measure of Surfaces.

41. The denominations of Square Measure are, square inch, square foot, square yard, and square perch or pole.

Table.

144 square* inches	make	1 square foot,
9 square feet		1 square yard,
$30\frac{1}{4}$ square yards		1 square perch or pole, American measure,
49 square yards		1 square perch or pole, Irish measure.

Rule.

Square poles, Irish measure	×	49	give square yards,
Square poles, American do.	×	$30\frac{1}{4}$	square yards,
Square yards	×	9	square feet,
Square feet	×	144	square inches.

Ex. 1. Reduce 245 square poles, Irish measure, to square inches. Ans. 15558480.

2. Reduce 120 square poles, American measure, to square inches. Ans. 4704480.

3. In 5 square feet, 120 square inches, how many square inches. Ans. 840.

Land Measure.

42. The denominations of Land Measure are acre, rood, and square pole or rod.

Table.

40 square poles	make	1 rood,
4 roods		1 acre,†
640 acres		1 square mile.

* A *square* is a figure which has four equal sides, each perpendicular to the adjacent ones. A *square inch* is a square, each of whose sides is an inch in length; a *square yard* is a square, each of whose sides is a yard in length, &c. The table of square measure is formed by multiplying each lineal dimension by itself; thus, a square foot is $=12\times12=144$ square inches, &c.

† In measuring land, surveyors use a *chain*, which is 4 perches in length, and is divided into 100 equal parts, called *links*. They also compute by chains and links, but exhibit the result in acres, roods, and perches. Ten square chains, or 100,000 square links, are an acre.

This is obviously a continuation of the table of square measure. By this and square measure, land, husbandmen and gardener's work, are measured,—boards, glass, plastering, tiling, flooring, and every dimension of length and breadth.

Rule.

Acres multiplied by 4 give roods,
Roods $\times$ 40 square poles or perches.

Ex. 1. Reduce 37 acres, 3 roods, 12 poles, to poles.
Ans. 6050.

2. Reduce 256 Irish acres, 15 poles, to yards.
Ans. 2007775.

3. How many square yards are there in 5604 acres?
Ans. 27123360.

4. In 561 acres of land, how many poles and yards?
Ans. 89760*P*. and 2715240*yds*.

5. How many poles are there in 997 acres, 2 roods, 10 poles? Ans. 135610.

Cubic or Solid Measure, or the Measure of Capacity.

43. The denominations of Cubic Measure are cubic yard, cubic foot, and cubic inch.

Table.

1728 cubic inches* make 1 cubic foot,
27 cubic feet 1 cubic yard.

This measure is used to estimate the quantity of stone or marble in blocks, or of timber in trees, &c.—40 feet of round timber, or 50 feet of hewn timber, make one load or ton—42 feet is equal to 1 ton of shipping—128 cubic feet (that is, a pile of wood, 8 feet long, 4 feet wide, and 4 feet high,) make a *cord* of wood.

* A cube is a body contained by six equal squares. Dice afford a familiar instance of this body. A *cubic inch* is a cube whose sides are each a square inch; a *cubic foot* a cube whose sides are each a square foot, &c. It may be remarked that 1728 is equal to $12\times12\times12$, and 27 equal to $3\times3\times3$.

Rule.

Cubic yards multiplied by	27	give cubic feet,	
Cubic feet	×	1728	cubic inches.

Ex. 1. In 60 cubic feet, how many cubic inches? Ans. 103680.

2. In 70 cubic yards, how many cubic feet? Ans. 1890.

3. In 30 cubic yards, 23 cubic feet, and 600 cubic inches, how many cubic inches? Ans. 607024.

4. In 2 tons, 12 feet, of hewn timber, how many cubic inches? Ans. 1057536.

5. In a cord of wood, or 128 cubic feet, how many cubic inches? Ans. 221184.

Liquid Measure.

44. The denominations of Liquid Measure are tun, pipe, hogshead, barrel, gallon, quart, pint, and gill.

4	gills, *gil.*	make	1 pint, *pt.*
2	pints		1 quart, *qt.*
4	quarts, or 8 pints		1 gallon, *gal.*
$31\frac{1}{2}$	gallons		1 barrel, *bbl.*
63	gallons		1 hogshead, *hhd.*
2	hogsheads		1 pipe or butt, *p.*
2	pipes, or 4*hhds.*		1 tun, *T.*

This is evidently a species of cubic measure, and is used to measure wine, spirits, beer, cider, &c. This measure, however, extends only to the gallons, all liquids being in general bought and sold by the gallon, which contains 231 cubic inches. For, in the measure of foreign wine, (and some domestic liquids) there are great varieties with respect to the number of gallons which the tun, pipe, &c. contain. A pipe of Madeira is 110 gallons—of Teneriffe and Barcelona 120*gals.*—of Sherry 130—of Port 138—of Lisbon 140. The hogshead of Claret is 58 gallons. A tun of animal oil is 252 gallons, and of vegetable oil 236 gallons. A tierce (the third part of a pipe) is 42 gallons, a punch is 84 gallons, and a barrel of beer in some places is 36 gallons, the gallon containing 282 cubic inches. In the United States, and especially in the state of New-York, the ordinary measure for all liquids is Wine Measure.

Reduction of Liquid Measure, Dry Measure, &c. is so easy, after what has gone before, that it is unnecessary to give particular rules for each kind.

Ex. 1. In 65 tuns, 3*hhds.* and 30*gals.* how many gallons? Ans. 16599.

2. In 7 gallons, how many gills? Ans. 224.

3. In 1 gallon, how many gills? Ans. 32.

4. How many gallons are there in 5 pipes of wine? Ans. 630.

5. In 36 gallons of wine how many pints? Ans. 288.

Dry Measure.

45. The denominations of this measure are pint, quart, gallon, peck, bushel, and chaldron.

Table.

2 pints (*pt.*)	make	1 quart, *qt.*
4 quarts		1 gallon, *gal.*
2 gallons, or 8 quarts		1 peck, *p.*
4 pecks		1 bushel,* *bush.*
36 bushels		1 chaldron of coal, *ch.*

This measure, which is another species of cubic measure, is used in measuring grain, seeds, salt, and various kinds of dry articles. In many places, however, these are bought and sold by weight.—For comparing the price of grain when sold by measure with its price when sold by weight, it may be useful to know that 57*lb.* Avoirdupois of good wheat, 55*lb.* of rye, 49*lb.* of good barley, or 38*lb.* of oats, shall be deemed equal to 1 bushel. This standard will not, however, apply in general to the grain in the United States. The gallon of Dry Measure contains 268$\frac{4}{5}$ cubic inches, and the bushel 2150$\frac{2}{5}$ cubic inches.

Ex. 1. How many pints are there in 19 bushels and 2 pecks of Canary seed? Ans. 1248.

2. How many bushels are there in 72 chaldrons of coal? Ans. 2592.

3. How many pints are there in 1 bushel of salt? Ans. 64.

* In England 8 bushels are equal to 1 quarter—5 quarters=1 wey—2 weys, or 10 quarters=1 last.

4. Reduce 7 bushels, 3 pecks, 2 quarts, and 1 pint, to pints. Ans. 501.

Time.

46. The denominations of Time are year, month, week, day, hour, minute, and second.

Table.

60 seconds, *sec.*	make	1 minute, *m.*
60 minutes		1 hour, *h.*
24 hours		1 day, *d.*
7 days		1 week, *w.*
52 weeks and 1 day, or 365 days		1 year, nearly.

The year is divided into 12 portions, called *calendar months*, the names of which are January, February, March, April, May, June, July, August, September, October, November, December. Of these, April, June, September, and November, have 30 days each, and the rest, except February, have 31 days each. In leap-years* February has 29 days—in common years 28 days. Hence a leap-year contains 366, and any other 365. The precise length of the year is found to be 365 days, 5 hours, 48 minutes, and 48 seconds: it is, therefore, 365 days, 6 hours, nearly.

Ex. 1. In 17 days, how many minutes? Ans. 24480.

2. Reduce 1 day, 4 hours, 12 seconds, to seconds. Ans. 100812.

3. How many days are there from the 8th of May till the 4th of July?

```
31 days in May.
 8
——
23  May,
30  June,
 4  July.
——
57 days,—Ans.
——
```

* Leap-years occur at intervals of 4 years, and may be known by dividing by 4 the number expressed by the last two figures in the number

4. How many days are there from the 8th of January, 1828, till the 12th of December in the same year? Ans. 339.

5. How many days are there between the 17th of March and the 25th of December? Ans. 283.

Division of the Circle.

47. The circumference of every circle is supposed to be divided into 360 equal parts called *degrees*—each degree is subdivided into 60 equal parts called *minutes*—each minute into 60 equal parts called *seconds*—each second into 60 equal parts called *thirds*. Hence the following

Table.

60 seconds, 60″	make	1 minute, 1′
60 minutes		1 degree, 1°
30 degrees		1 sign,
12 signs, or 360 degrees		1 great circle.

This is used by astronomers, navigators, mathemeticians, &c. Suppose the circumference of the earth to be divided into 360 degrees, 1 degree contains 60 geographical, or 69½ American miles.

of the year, according to the Christian era: if there be no remainder, it is leap-year; otherwise the remainder shows how many years it is after leap-year. To this there is one exception, as the exact centuries are not leap-years, except when the number of centuries are divisible by 4 without remainder. Thus, the year 1824 was leap-year, because 24 is divisible by 4; but 1827 is the third year after leap-year, because 3 remain when 27 is divided by 4. Also the year 2000 is leap-year, but 1900 not, as 20 is divisible by 4, but 19 is not.

* Degrees, minutes, seconds, &c. are marked thus: 41° 24′ 54″ 21‴, and is read, 41 degrees, 24 minutes, 54 seconds, and 21 thirds. The reason of these marks being employed will appear evident from the consideration that minutes, seconds, &c. are only abbreviated expressions for *first* minutes, or minutes of the *first order*, *second* minutes, &c.; *minutes*, in each instance, signifying *small parts*. It may be proper to remark, that the *circumference* of a circle is the line which contains it,—that all straight lines drawn from the centre to the circumference are equal,—that any of these lines is called the radius,—and that a line drawn through the centre and terminated both ways by the circumference, is called a diameter.

Ex. 1. In 15 degrees how many seconds? Ans. 54000.
2. Reduce 41 degrees, 24 minutes, and 54 seconds to seconds. Ans. 149094.
3. Reduce 360 degrees to seconds. Ans. 1296000.

Miscellaneous Table.

48. Paper, parchment, books, &c.

12 articles of any kind	make	1 dozen,
12 dozen		1 gross,
24 sheets of paper		1 quire,
20 quires		1 ream,
2 reams		1 bundle,
10 reams		1 bale.
5 skins of parchment		1 roll.
72 words in common law		1 sheet,
90 do. in Chancery		1 sheet.

Folio is the largest size of books, of which

Folio	2 leaves,	or 4 pages	=1 sheet.
Quarto, 4to.	4	8	=1 do.
Octavo, 8vo.	8	16	=1 do.
Duodecimo, 12mo.	12	24	=1 do.
Octodecimo, 18mo.	18	36	=1 do.

From the preceding abstract of the principal tables now in use, it will appear that the system of our weights and measures is extremely perplexed and intricate, and that there is great necessity for the interference of Congress to introduce that simplicity and uniformity, which would contribute to the ease and fairness of mercantile transactions. It is evident that in the same nation the same system of weights and measures should be universally adopted. Such a change would inevitably have considerable difficulties opposed to it at first; as people would with much reluctance, and no doubt with inconvenience, lay aside the use of the weights and measures to which they had long been accustomed: and such in fact has been the case in France, where a new system* was introduced after the Revolution in that country.

* In this system the divisions are all adapted to the decimal system of notation. Thus, in Long Measure, the standard from which all the

REDUCTION ASCENDING.

To reduce a quantity from a lower to a higher denomination.

49. Rule. Divide the number expressing the quantity by the number which shows how many of the denomination in which it is make *one* of the higher denomination: the quotient will be of the higher denomination, and if there be any remainder, it will be of the lower.

When there are intermediate denominations between that in which the quantity is given and that to which it is to be reduced, it is generally better to reduce it by successive steps; first to one or more of the intermediate denominations, and then to the required denomination.

Method of Proof.

To the answer found by this rule, apply the rule for Reduction Descending, and if the result be the same as the given number, the work is correct. Hence Reduction Descending may be proved in a similar manner, by applying the rule for Reduction Ascending.

Ex. 1. In 350600 cents how many eagles and dollars?

1|00)3506|00

1|0)350|6 dollars.

350 eagles and 6 dollars.

other measures are derived, is the ten-millionth part of an arc of the meridian extending from the equator to the pole, and is called a *metre*—ten metres constitute a measure called the *decametre*—ten decametres a *hectometre*, &c.: and the tenth part of a metre is called a *decimetre*—the tenth part of a decimetre a *centimetre*, &c. The adaptation of this system to the decimal notation, gives it the advantage of facilitating in an extreme degree almost all the mercantile operations in arithmetic.

2. Reduce 83918 farthings to pounds.

$$\begin{array}{l} 4)83918 \text{ farthings.} \\ \hline 12)20979\tfrac{2}{4} \\ \hline 2\not{0})174\not{8} \quad 3\tfrac{2}{4} \\ \hline \pounds 87 \quad 8 \quad 3\tfrac{2}{4}d. \end{array}$$

In this example, the farthings are divided by 4, because in the same sum there are 4 times as many farthings as there are pence: for a similar reason the pence are divided by 12 to reduce them to shillings, and the shillings thus found by 20 to reduce them to pounds. Hence it appears that 83918 farthings are equivalent to 20979 pence, with two farthings or a halfpenny; to 1748 shillings and 3 pence halfpenny, or finally, to £87 8 3$\tfrac{2}{4}$. The operation is proved by reducing the operation to farthings, thus:

$$\begin{array}{rrr} \pounds & s. & d. \\ 87 & 8 & 3\tfrac{2}{4} \\ 20 & & \\ \hline 1748 & & \\ 12 & & \\ \hline 20979 & & \\ 4 & & \\ \hline 83918 & \text{farthings,—}\textit{proof.} & \end{array}$$

3. Reduce 591241*lb.* to tons.

$$\begin{array}{rrrr} \textit{lbs.} & & & \\ 4)591241 & & & \\ \hline 7)147810 & 1 & & \\ \hline 4)21115 & 21 & & \\ \hline 2\not{0})527\not{8} & 3 & 21 & \\ \hline T.\ 263 & 18 & 3 & 21 \end{array}
\qquad
\begin{array}{rrrr} T. & cwt. & qrs. & lb. \\ 263 & 18 & 3 & 21 \\ 20 & & & \\ \hline 5278 & & & \\ 4 & & & \\ \hline 21115 & & & \\ 28 & & & \\ \hline 168941 & & & \\ 42230 & & & \\ \hline 591241 & \textit{lbs.} & & \end{array}$$

Proof, 591241 *lbs.*

In this example the pounds are divided by 28 (or, which is equivalent, by 4, and the quotient by 7) to reduce them to quarters—the quarters by 4, to reduce them to hundreds—and the hundreds by 20 to reduce them to tons. From the operation it appears that 591241*lbs.* are equivalent to 21115*qrs.* 21*lbs.*; or to 5278*cwt,* 3*qrs.* 21*lb.*; or to 263 tons, 18*cwt.* 3*qrs.* 21*lb.*

4. In 37030 mills how many dollars and cents?

Ans. $37·03.

5. In 2300 pence, English money, how many pounds?

Ans. £9 11 6*d.*

6. Reduce 1785*dwts.* to pounds. Ans. 7*lb.* 5*oz.* 5*dwts.*

7. In 12960 grains of Guinea gold how many ounces?

Ans. 27.

8. In 14769 ounces of Swedish iron, how many hundredweight? Ans. 8*cwt.* 27*lb.* 1*oz.*

9. In 21444 drams of Persian silk, how many pounds?

Ans. 83*lb.* 12*oz.* 4*dr.*

10. The battering-ram which was employed by Titus to demolish the walls of Jerusalem weighed, according to Josephus, 100,000*lb.*; how many tons, &c. do they contain?

Ans. 44 tons, 12*cwt.* 3*qrs.* 12*lb.*

11. How many tons does the large bell of Moscow weigh, its weight being 432000*lb.*?

Ans. 192 tons, 17*cwt.* 0*qrs.* 16*lb.*

12. How many tons, &c. does the enormous pedestal of reddish granite, a *single stone*, on which stands the equestrian statue of Peter the Great, at Petersburg; its weight being 3,200,000*lb.*? Ans. 1428 tons, 11*cwt.* 1*qr.* 20*lb.*

13. In 159022 grains, how many pounds, ounces, drams, scruples, and grains? Ans. 27℔ 7℥ 2ʒ 1℈ 2*gr.*

14. In 3 dozen napkins, each containing 20 nails, and 1 dozen tablecloths, each containing 38 nails, how many yards?

Ans. 73*yds.* 2*qrs.*

15. Hannibal, the Carthaginian general, set out from New Carthage (now Carthagena) in Spain, about 217 years, B. C. to invade Italy;—and the distance from New Carthage to the plains of Italy is computed at 63,360,000 inches, how many miles are in this space? Ans. 1000.

16. The ruins of the city of London, in the great fire in the year 1666, are computed to have covered 69760 square poles of ground, from the Tower, by the Thames side, to the Temple Church, and from the Northeast gate, along the

city-wall, to Holborn-bridge. How many acres are in that quantity? Ans. 436.

17. The base of the great pyramid of Egypt covers 1760 square poles, how many acres are in this quantity?
Ans. 11.

18. In 96,000,000 pints of beans, how many bushels?
Ans. 1,500,000.

19. In 607024 cubic inches how many cubic yards?
Ans. 30 cubic yds. 23 cubic feet, and 600 cubic inches.

20. In 126 pint-bottles of wine, how many hogsheads?
Ans. 3.

21. It is said that 20,160,000 pints of Port wine are annually exported from Oporto in Portugal; how many pipes are in that quantity? Ans. 20,000.

22. The island of Madeira, which belongs to Portugal, produces 8064000 bottles of wine annually, each containing 3 pints and a half; how many pipes might be filled with that quantity? Ans. 28000.

23. How much time, in the course of 40 years, does a person who rises at 5 o'clock in the morning gain over another who continues in bed till 7, supposing them both to go to rest at the same hour at night?
Ans. 3 years, 121 days, and 16 hours.

24. The fortunate war which America maintained with England continued nearly 252460800 minutes, how many years did it last? Ans. 8.

Questions.

What is reduction?
How many kinds, and what are they called?
What is reduction descending?
What is reduction ascending?
How is the operation of reduction descending performed? Repeat the rule.
What are the denominations of Federal money?
Repeat the table, and also the particular rule for reducing this money, from a higher to a lower denomination.
What are the denominations of English money?
Repeat the rule, &c. as in Federal money.
What are the denominations of Troy weight?
Repeat the table, &c.
For what purpose is Troy weight used?
What are the denominations of Avoirdupois weight?

Repeat the table, &c.

What are the denominations of Apothecaries weight?

Repeat the table, &c.

What are the denominations of Cloth measure?

Repeat the table, &c.

What are the denominations of Long measure?

Repeat the table.

What are the denominations of Square measure? and repeat the table.

What are the denominations of Land measure? and repeat the table.

What are the denominations of Cubic measure? and repeat the table.

What are the denominations of Liquid measure? and repeat the table.

What are the denominations of Dry measure? and repeat the table.

What are the denominations of Time? and repeat the table.

Repeat the table for the division of the circle.

By whom is this table used?

Repeat the rule for performing the operation of reduction ascending.

How is reduction descending proved?

How is reduction ascending proved?

Exercises in Reduction ascending and descending.

Ex. 1. During the year 1826, there were made at the mint in the United States, 18069 half and 7602 quarter eagles, and 4004180 half dollars. The gold, silver, and copper coinage, made from the commencement of the institution to the 31st of December, 1825, was $25390966. How much is the grand total, allowing there was $17161 in cents, made in 1826? Ans. $27502462.

2. In the mint in London there are eight coining presses, which, with a child to supply each, strike 19000 coins in an hour. Now, if they were employed 12 hours each day, for 313 days, in coining halfpence, what would be the number and the value of all the pieces coined during that time? Ans. *number*, 71364000; *value*, £148675.

3. The quantity of linen imported into the United States from Ireland, in 1806, was two millions, six hundred and

seventy-five thousand, six hundred and nineteen yards. How many miles in length was the whole?

Ans. 1520*m.* 1*f.* 36*p.* and 1*yd.*

4. In the city of Pekin there are said to be seven bells, each weighing 120000*lb.*; and in Moscow there is one bell which weighs 127836*lb.*; another which weighs 288000*lb.*; and a third which weighs 432000*lb.* Required the weight of each in tons. Ans. 53*t.* 11*cwt.* 1*qr.* 20*lb.*; 57*t.* 1*cwt.* 1*qr.* 16*lb.*; 128*t.* 11*cwt.* 1*qr.* 20*lb.*; and 192*t.* 17*cwt.* 16*lb.*

5. Suppose one person to lie in bed nine hours each day at an average, and another only six hours and a half; and suppose the latter to employ the time thus gained in reading and study, for 48 years; to how many years' study, of 12 hours each day, would the entire time gained be equivalent.

Ans. 8 years, 121 days, and 8 hours.

6. The whole surface of the earth contains 196649494 square miles, and of these Europe is supposed to contain 4456065. Required the number of acres contained in each. Ans. 125855676160, and 2851881600.

7. In what time would a body move from the Earth to the Moon, at the rate of thirty-one miles per day; the mean distance being 238545 American miles?

Ans. 21 years, and 30 days.

8. In what time would a body moving with the velocity of sound, (which is known to move at the rate of 1130 feet per second,) pass from the Earth to the Sun, the distance being ninety-five millions of miles?

Ans. 14 years, 27 days, 15 hours, 50 min. $5\frac{350}{1130}$ sec.

9. In how long time would a cannon ball, with the velocity of 1960 feet per second, move from the Sun to Uranus?

Ans. 155 years, 224 days, 5 hours, 46 min. $7\frac{680}{1960}$ sec.

10. By the latest measurements, the Earth's mean diameter is found to be 7920 miles. How many yards, feet, and inches are contained in it?

Ans. 13924680 yds.; 41774040 ft.; 501288480 inches.

11. The number of penny-pieces which have been lately coined, and actually circulated in England, amounts to 40000000, each weighing exactly an ounce; of course, the halfpence of the same coinage weigh half an ounce, the farthings a quarter of an ounce. How many tons, &c. are in the above number of penny-pieces, and how many pounds sterling do they contain? Ans. 1116 *tons,* 1*cwt.* 1*qr.* 20*lb.* weight; £166666 13*s.* 4*d.* value.

12. The United States of America were declared *Free and Independent* 1609403328 seconds ago this present year, 1827; how many years have intervened since, allowing the year to consist of 365 days, 5 hours, 48 minutes, and 48 seconds? and in what year did that glorious event take place? Ans. 51 years ago; and in the year 1776, the period in which it happened.

CHAPTER III.

Compound Addition, Subtraction, Multiplication, and Division.

49. When the numbers to be added express quantities of the same kind, but of different denominations, the operation is called *compound addition.*

RULE.* Arrange the given quantities so that those in each column may be of the same denomination. Add the numbers of the lowest denomination together; reduce their sum to the next higher denomination; set the remainder below the column added, and carry the quotient to the next. Proceed thus with all the other denominations, except the highest, which is to be added in the same manner as in addition of whole numbers.

Either of the first two methods of proof given in the addition of whole numbers, (usually called Simple Addition,) may be employed in Compound Addition.

Federal Money.

Addition of Federal Money (if there are no fractions) is performed in the same manner as addition of whole num-

* The reason of this rule is evident, from what has been said in addition of whole numbers: for instance, the addition of English money, as 1 in the pence is equal to 4 in the farthings; 1 in the shillings, to 12 in the pence; and in the pound, to 20 in the shillings; therefore carrying as directed, is nothing more than providing a method of placing the money arising from each column properly in the scale of denominations: and this reasoning will hold good in the addition of compound numbers of any denomination whatever.

bers; but it must be observed, that the cents are separated from the dollars by placing a point between them.*

Ex. 1. Add together the following sums: \$35; \$76.75; \$109.06; and \$300.29.

Here, 9 and 6 are 15, and 5 are 20, set down 0, and carry 2; again, 2 and 2 are 4, and 7 are 11, set down 1, and carry 1; then, 1 and 9 are 10, and 6 are 16, and 5 are 21, set down 1, place a point between the cents and the dollars, and carry 2; proceed as in addition of whole numbers, and the amount, or sum will be 521 dollars, and 10 cents.

	\$ 35
	76.75
	109.06
	300.29
sum,	\$521.10

2. Add together the following sums: $\$9.12\frac{1}{2}$; $\$25.74\frac{3}{4}$; and $\$13.06\frac{1}{4}$.

Here 1 fourth and 3 fourths are 4 fourths, and $\frac{1}{2}$, or 2 fourths, are 6 fourths; which divided by 4, the quotient is 1, and the remainder 2; this remainder, 2 fourths or $\frac{1}{2}$, is set down, and the quotient figure 1 is added to the units of the cents.

$\$ 9.12\frac{1}{2}$
$25.74\frac{3}{4}$
$13.06\frac{1}{4}$
$\$47.93\frac{1}{2}$

	\$	cts.		\$	cts.		\$	cts.	m.
Ex. 3.	79	75	Ex. 4.	191	$19\frac{1}{2}$	Ex. 5.	3	06	7
	80	07		397	$89\frac{3}{4}$		5	17	3
	17	11		170	$08\frac{1}{4}$		1	05	4
	63	00		9	$78\frac{1}{4}$		14	19	8
		19			$63\frac{3}{4}$		327	09	7
Ans.			Ans.			Ans.			

Ex. 6. What is the sum total of 33 dollars 13 cents, 676 dollars 33 cents, 171 dollars 19 cents, and 184 dollars $37\frac{1}{2}$ cents? Ans. $\$1065.02\frac{1}{2}$.

Ex. 7. What is the sum of 18 cents, $37\frac{1}{2}$ cents, $\$1.18\frac{3}{4}$, 75 cents, $87\frac{1}{2}$ cents, $6\frac{1}{4}$ cents, $\$3.31\frac{1}{4}$, $43\frac{3}{4}$ cents, and 3 dollars? Ans. \$10.18.

8. Bought an Arithmetic for $37\frac{1}{2}$ cents; a slate for $18\frac{3}{4}$ cents; a writing-book for $12\frac{1}{2}$ cents; a ciphering-

* When there are fractional parts of a cent, such as $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, find their amount in fourths, counting $\frac{1}{2}$ as 2 fourths, divide their sum by 4; the quotient will be cents, and the remainder will be fourths of a cent, which set down, and carry the quotient figure to the units of cents; then proceed as in addition of whole numbers.

book for 25 cents; an English grammar for 37$\frac{1}{2}$ cents; a Geography for 37$\frac{1}{2}$ cents; an Atlas for 50 cents; Walker's Dictionary for 62$\frac{1}{2}$ cents; and an English Reader for 31$\frac{1}{4}$ cents. What is the cost of the whole? Ans. \$3.12$\frac{1}{2}$.

9. A man borrowed a certain sum of money, and paid \$376.37$\frac{1}{2}$; the sum left unpaid was \$127.37$\frac{1}{2}$. What was the original debt? Ans. \$503.75.

10. Laid out at market, in beef \$1.37$\frac{1}{2}$, in mutton 62$\frac{1}{2}$ cents, in lamb 87$\frac{1}{2}$ cents, in veal 50 cents, in fowl \$1.12$\frac{1}{2}$, in vegetables 18$\frac{3}{4}$ cents. What was expended in all? Ans. \$4.68$\frac{3}{4}$.

11. Expended in gloves \$1.12$\frac{1}{2}$, in stockings \$3.87$\frac{1}{2}$, in lace \$12.37$\frac{1}{2}$, in cambric \$25.50, in muslin \$6.75, in silk \$15.62$\frac{1}{2}$, and in shoes \$1.62$\frac{1}{2}$. What was the the whole sum spent? Ans. \$66.87$\frac{1}{2}$.

12. Bought a cloak for \$21, a shawl for \$29.35, a hat for \$15.75, and sundry articles for \$17.75. What is the whole cost? Ans. \$83.85.

English Money.

	£	s.	d.
Ex. 1.	39	18	7$\frac{3}{4}$
	51	12	4$\frac{1}{2}$
	79	19	10$\frac{1}{2}$
	8	7	11$\frac{1}{4}$
	43	13	9$\frac{3}{4}$
	375	16	10$\frac{3}{4}$
Sum,	599	9	6$\frac{1}{2}$*

In this example, the sum of the farthings is 14; which being divided by 4, the farthings in a penny, the quotient is 3 pence; and the remainder, 2 farthings, or a half-penny, is set down. The quotient, 3, is then added with the pence; the sum, 54, being divided by 12, the pence in a shilling, the quotient is 4 shillings, with a remainder of 6 pence, which is set down. The quotient, 4, is then added with the units of the shillings; the sum is 39, of which the latter figure is set down, and the tens being carried to the tens of the shillings, the sum is 8; which being divided by 2, because 2 tens, or 20 shillings, make a pound, the

* The proof is left to exercise the learner; and it will be proper to require him to perform it, not only in this example, but in all the exercises in this rule.

quotient is 4, which is added with the pounds, as in Simple Addition.*

In this example the halfpence amount to 5, or 2 pence halfpenny. In such examples, where all the fractional parts are halfpence, it is easier to call each a halfpenny than 2 farthings.

	£	s.	d.
2.	15	3	8½
	31	16	7½
	94	13	8½
	55	12	11½
	37	11	9½
Sum,	234	18	9½

When the columns are very long, the work becomes heavy and laborious; and therefore, in such a case, the given quantities may be separated into two or more divisions, as is suggested in page .

	£	s.	d.		£	s.	d.
3.	44	10	6	4.	199	10	11¾
	17	18	9		70	19	9½
	14	9	0		146	3	4
	7	3	10		7	9	6¼
	129	14	6			2	8¾
Ans.				ns.			

5. Add together £71 13 4, £95 14 9, £31 16 3, £20, £13 0 3, and £1600. Ans.

6. Required the sum of £13, 11¼d., 3s. 9d., £100, and 19s. and 11¾d. Ans.

* *Pence Table.*

d.	s.	d.	d.	s.	d.	d.	s.	d.	d.	s.	d.
12=	1	0	40=	3	4	72=	6	0	100=	8	4
20=	1	8	48=	4	0	80=	6	8	108=	9	0
24=	2	0	50=	4	2	84=	7	0	110=	9	2
30=	2	6	60=	5	0	90=	7	6	120=	10	0
36=	3	0	70=	5	10	96=	8	0	130=	10	10

This table has been inserted, lest some teachers should consider the want of it an imperfection. It seems better, however, not to impose upon the learner the labour of committing it to memory, except perhaps a small part at the beginning.

Troy Weight.

	lb.	*oz.*	*dwt.*	*gr.*
Ex. 1.	3674	3	15	22
	3017	6	16	19
	310	11	19	23
	42	10	7	1
	763	2	11	13
	7808	11	11	6

In adding up the columns of grains, we find the sum to be 78, which being divided by 24 to bring it into pennyweights, is equivalent to 3 pennyweights and 6 grains over; the 6 is put down, and the 3 is carried to the column of pennyweights; then these being added together, the sum is found to be 71, which, reduced into ounces, is equivalent to 3 ounces and 11 pennyweights; we put down 11, and carry 3 to the column of ounces; then adding the ounces, the sum is found to be 35, which, by dividing, gives 2 pounds 11 ounces, we put down 11, and carry the 2 to the pounds and proceed as in addition of whole numbers.

	lb.	*oz.*	*dwts.*	*gr.*
Ex. 2.	19	3	10	17
	76	1	9	23
	19	11	19	21
	8	10	13	11
	78	9	16	20
Ans.				

	lb.	*oz.*	*dwts.*	*gr.*
Ex. 3.	196	10	12	15
	796	0	7	0
	340	11	19	20
	673	10	17	23
	7	3	16	16
Ans.				

Ex. 4. Required the sum of 48*lb.* 11*oz.* 18*dwt.* 21*gr.*; 42*lb.* 10*oz.* 14*dwt.*; 40*lb.* 9*oz,* 16*dwt.* 20*gr.*; 36*lb.* 8*oz.* 15*dwt.* 22*gr.*; 38*lb.* 10*oz.* 10*dwt.*; and 53*lb.* 17*dwt.* 13*gr.*
Ans. 261*lb.* 4*oz.* 13*dwt.* 4*gr.*

5. Required the sum of 35*lb.* 11*oz.*; 17*lb.* 19*dwt.*; 30*lb.* 1*oz.* 20*gr.*; 17*lb.* 4*gr.*; and 165*lb.* 11*oz.*
Ans. 266*lb.*

Avoirdupois Weight.

In this example the sums of the pounds, quarters, and hundreds, are respectively divided by 28, 4, and 20, the remainders set down below their respective columns, and the quotients carried to the next columns respectively. The hundreds may be added as shillings in addition of English money, the divisor being the same in both cases.*

	T.	cwt.	qrs.	lb.
Ex. 1.	35	16	0	20
	42	14	2	18
	18	9	1	16
	17	18	3	7
	31	5	3	19
	45	12	2	5
Sum,	191	17	2	1

Ex. 2.			Ex. 3.			Ex. 4.		
cwt.	qrs.	lb.	lb.	oz.	dr.	T.	cwt.	qrs.
76	3	15	19	12	12	17	14	3
14	1	27	21	9	13	18	19	1
35	2	21	16	15	14	78	12	2
16	1	19	23	10	10	99	13	0
10	0	0	18	14	15	30	10	1
70	3	24	21	11	11	19	19	3
Ans.			Ans.			Ans.		

Ex. 5. Add together 17 hundredweight, 3 quarters, 14 pounds; 13 hundredweight, 1 quarter, 21 pounds; and 21 hundredweight, 2 quarters 19 pounds.

Ans. 52*cwt.* 3*qrs.* 26*lb.*

6. Add together 17 pounds, 15 ounces; 6 pounds, 6 ounces; and 18 pounds, 9 ounces, 13 drams.

Ans. 42*lb.* 14*oz.* 13*dr.*

7. Add together 17 tons, 13 hundredweight, 3 quarters; 16 tons, 17 hundredweight, 1 quarter; and 396 tons, 3 hundredweight. Ans. 430*T.* 14*cwt.*

* This illustration will be sufficient for the various examples in the other weights and measures, which differ only in the value of the divisors.

Apothecaries' Weight.

	℔	℥	ʒ		ʒ	℈	gr.
Ex. 1.	21	11	4	Ex. 2.	3	1	19
	19	9	3		1	0	10
	17	10	7		0	2	11
	18	3	6		2	1	0
Ans.	77	11	4	Ans.	8	0	0

Ex. 4. An apothecary made a composition of five ingredients; the first weighed 3℔ 7℥; the second 11℥ 7ʒ 13*gr.*; the third 7℔ 2℈; the fourth 1℔ 3ʒ 1℈; and the fifth 5℔ 5℥ 2ʒ 1℈ 7*gr.* What was the weight of the whole?
Ans.

Cloth Measure.

	yds.	*qrs.*	*n.*		*yds.*	*qrs.*	*n.*	*in.*
Ex. 1.	376	3	3	Ex. 2.	319	1	2	2
	196	1	1		167	2	3	1
	764	3	2		763	1	0	2
	145	1	3		345	3	2	1
Ans.	1483	2	1	Ans.	1596	1	1	$1\frac{1}{2}$*

Ex. 3. A merchant bought 4 parcels of cloth; the first contained 131*yds.* 1*qr.* 2*n.*—the second 96*yds.* 2*qrs.* 3*n.*—the third 120*yds.* 3*qrs.* 3*n.*—and the fourth 75*yds.* 1*qr.*: how many yards, &c. were in the whole? Ans. 424*yds.* 1*qr.*

4. Bought 4 pieces of linen: the first contained 23*yds.* 2*qrs.* 1*n.*—the second 24*yds.* 2*qrs.*—the third 25*yds.* 3*qrs.* 3*n.*—and the fourth 27*yds.* 2*qr.*: how many yards were in the whole? Ans. 101*yds.* 2*qrs.*

* In adding the column of inches, we find the sum to be 6, which is equivalent to 24 quarters of an inch, and 9 quarters are equal to a nail: hence 24 is divided by 9, the quotient is 2; and the remainder, 6 quarters, is equal to an inch and a half.

Long Measure.

Ex. 1.				2.				3.		
l.	*m.*	*f.*		*m.*	*f.*	*p.*		*yd.*	*ft.*	*in.*
75	2	7		13	3	39		16	2	11
14	1	3		37	2	16		13	1	9
7	1	2		14	1	22		76	0	10
87	0	3		17	7	10		18	1	9
18	1	0		90	0	9		9	2	11
Sum, 203	0	7	Sum,				Sum,	135	1	2

4. Suppose the distance from New-York to Newark to be 8 miles, 7 furlongs, 39 poles; from thence to Elizabeth-town, 9 miles and 1 pole; from thence to Woodbridge, 11 miles, 6 furlongs, and 35 poles; from Woodbridge to New-Brunswick, 4 miles, 1 furlong, 5 poles; from thence to Princeton, 10 miles, 5 furlongs; from Princeton to Trenton, 17 miles, 3 furlongs; and from Trenton to Philadelphia, 22 miles. What is the distance from New-York to Philadelphia? Ans. 84 miles.

5. The distance from Philadelphia to Carlisle is 119 miles, 7 furlongs, 39 poles; from Carlisle to Greensburg, 148 miles, 1 pole; and from Greensburg to Pittsburg, 32 miles. What is the distance from Philadelphia to Pittsburg? Ans. 300 miles.

Square and Land Measure.

Ex. 1.			2.			3.		
s.yd.	*s.ft.*	*s.in.*	*acres.*	*r.*	*p.*	*acres.*	*r.*	*p.*
76	7	129	736	2	17	396	1	10
19	8	136	197	1	29	196	2	30
14	2	129	317	3	39	703	3	39
73	4	100	296	0	27	497	0	18
19	6	96	103	2	19	123	3	25

4. A surveyor having measured four farms of land, found one to contain 97 acres, 3 roods, 25 poles: another, 119 acres, 2 roods, 16 poles; the third, 220 acres; and the fourth, 137 acres, 1 rood, 39 poles. How many acres were surveyed? Ans. 575 acres.

5. A painter agreed to paint four rooms in my house; the first of which measured 39 square yards, 7 square feet, 37 square inches; the second, 45 square yards, 8 square feet, 130 square inches; the third, 50 square yards, 6 square feet, 140 square inches; and the fourth, 24 square yards, 3 square feet, 125 square inches. How many square yards, &c. are to be paid for? Ans. 161 square yards.

Questions.

When is compound addition used?
How do you arrange the quantities to be added?
Repeat the rule of operation.
How is compound addition proved?

*Exercises in Compound Addition.**

Ex. 1. Bought a gold watch for \$120; chain, key, &c. for \$42 25; a horse and gig for \$256 75; and a suit of black clothes for \$119 25. What was the amount which I laid out? Ans. \$538 25.

2. The following goods, at their respective value, were imported into the United States, in the year 1822: woollen goods from England, valued at \$12200196.75; cotton goods from England and China, \$10300000.37½; silk goods from France, China, and British East Indies, \$6800000 62½; and linen goods from England, Scotland, Ireland, and Germany, \$4100009.25. What is the entire amount?

Ans. \$33400207.

3. The national debt of England,

		£	s.	d.
At the Revolution, in	1689, was	1054925	12	6
At the peace of Ryswick,	1697,	21515742	7	6
At the peace of Utrecht,	1714,	53681075	0	0
At the peace of Aix-la-Chapelle,	1758,	78293312	19	3
At the peace of Paris,	1763,	183159275	0	9
At the peace of Versailles, after the American war,	1783,	238232247	19	11
At the peace of Amiens,	1802,	499752073	0	1
Amount of the debt,	1813,	600000000	0	0
Estimated amount, on 5th Jan.	1827,	900000000	0	0

* Those parts of Compound Addition, (Liquid measure, Dry measure, &c.) are performed without difficulty, by the application of the same principles on which the others depend.

Required the sum of these several debts.

Ans. £2575788652.

4. The following are the quantities, and values of linen exported from Ireland to America in the undermentioned years: required the value of the whole, and the number of American miles contained in the entire number of yards.

		yards.		£	*s.*	*d.*
In 1798	-	2361483	-	183343	4	0
1799	-	1376382	-	106818	16	1
1800	-	1156467	-	89948	11	11
1801	-	2519575	-	248762	2	6
1802	-	1089223	-	106671	18	6
1803	-	1873423	-	184180	18	6
1804	-	2258176	-	222034	5	9
1805	-	2221606	-	209404	6	2
1806	-	2675619	-	309525	9	9
1807	-	1657446	-	191413	3	6

Ans. Value, £1853102 16*s.* 8*d.*; length, 10903 miles, 21 poles, and 4½ yards.

5. The following are the several quantities of land contained in the Royal Forests, in England: New Forest, 66942 acres, 3 roods, 26 poles; Dean Forest, 23015 acres, 3 roods, 29 poles; Aliceholt and Woolmer, 8694 acres, 1 rood, 31 poles; Whittlewood, 4850 acres, 3 roods, 32 poles; Whichwood, 3709 acres, 3 roods, 5 poles; Waltham, 3278 acres, 3 roods, 2 poles; Salcey, 1847 acres, 23 poles; Sherwood, 1466 acres, 3 roods, 10 poles; Bere, 926 acres, 2 roods, 13 poles; Rockingham, 860 acres, 3 roods, 23 poles. Required the sum. Ans. 115594 acres, 34 poles.

5. A gentleman ordered a service of plate from his silversmith; and on receiving his bill, he finds that he had dishes and covers weighing 45*lb.* 9*oz.* 12*dwts.*; plates weighing 70*lb.* 7*oz.* 16*dwts.*; spoons of different sizes, and ladles, 24*lb.* 9*oz.* 12*dwts.*; waiters, 15*lb.* 10*oz.*; salts and castors, 4*lb.* 4*oz.* 3*dwts.*; candlesticks, 19*lb.* 11*oz.* 17*dwts.*; and sundry smaller articles, 5*lb.* 3*oz.* 10*dwts.* What is the weight of silver he will have to pay for?

Ans. 186*lb.* 8*oz.* 10*dwts.*

6. A merchant imported 8 tuns of Claret; 12 tuns, 1 hogshead, 9 gallons of Port; 4 tuns, 1 pipe of Sherry: 1 hogshead, 12 gallons, 3 quarts of Lisbon. How many tuns, &c. were imported in the whole?

Ans. 25 tuns, 21 gal. 3 qrts.

7. The latitudes of the following places on the coast of the United States of America, are;

	Latitude.		
Entrance of St. Croix River	45°	7′	13″
Portland Light House	43	39	01
Portsmouth do.	43	04	00
Salem	42	32	59
Boston	42	23	00
Newport	41	29	10
New Haven	41	10	48
New-York City	40	42	00
Philadelphia	39	57	08
Washington City	38	53	01
Baltimore	39	23	12
Charleston, S. C.	32	45	55
Savannah	32	02	10

Required the whole sum of those degrees, minutes, and seconds. Ans. 525° 49′ 37″.

8. Bought, 130 bushels of corn of one man; 190 bushels, 3 pecks, 3 quarts, of another; 300 bushels 1 peck, 7 quarts, of a third; 75 bushels, 1 quart, of a fourth. How many were bought in all? Ans. 696 bushels, 1 peck, 3 quarts.

9. Add together the following quantities: 796 cubic inches; 18 cubic feet, 1200 cubic inches; 19 cubic feet, 375 cubic inches; and 20 cubic yards, 9 cubic feet, and 1500 cubic inches. Ans. 21 c. yds., 21 c. ft., 415 c. in.

10. The following are the periods in which the primary planets perform their revolutions round the Sun:

	days.	hrs.	min.	sec.
Mercury	87	23	15	44
Venus	224	16	49	11
The Earth	365	5	48	51
Mars	686	23	30	36
Vesta	1335	4	55	12
Juno	1590	23	57	7
Ceres	1681	12	56	10
Pallas	1681	17	00	58
Jupiter	4332	14	18	41
Saturn	10758	23	16	34
Uranus	30688	17	6	02

Required their sum. Ans. 53434 days, 14 hours, 55 minutes, 6 seconds.

Compound Subtraction.

50. When the given numbers express quantities of the *same kind*, but of *different denominations*, the process is termed *compound subtraction*.

51. Rule. Place the less number below the greater,* so that the numbers in each column may be of the same denomination. Thus, beginning with the lowest denomination, subtract, if possible, each number in the lower line from that which stands above it. But when this cannot be done, subtract the number in the lower line from an unit of the next higher denomination; to the remainder add the upper number, for the true remainder, and carry one to the next higher number in the lower line.

Proceed thus with all the denominations, except the highest, in which the work is to be performed as in subtraction of whole numbers.

Federal Money.

Here, as $\frac{3}{4}$ of a cent is greater than $\frac{1}{2}$, it is taken from one cent, and the remainder ($\frac{1}{4}$) being added to $\frac{1}{2}$, or $\frac{2}{4}$, the sum ($\frac{3}{4}$) is set down, and one cent is then carried to five cents; and then proceed as in whole numbers.

		$	cts.
Ex. 1.	From	76	$37\frac{1}{2}$
	Take	48	$75\frac{3}{4}$
	Rem.	27	$61\frac{3}{4}$

Here the subtraction is performed as in whole numbers, and the remainder is 7 dimes, 4 cents, and 7 mills; or 74 cents and 7 mills.

		$	d.	c.	m.
Ex. 2.	From	19	7	3	2
	Take	18	9	8	5
	Rem.	00	7	4	7

* It is more usual, and generally more convenient, to set the less number below the greater in subtraction: it is by no means essential, however. And the pupil should be accustomed, both in Simple and Compound Subtraction, to subtract *downward* as well as *upward*, that he may be able to do so, when it may happen in complex operations, that the numbers are so arranged. The methods of proof, and the principles on which the operations depend, are the same as in Simple Subtraction.

In this example, the cents are distinguished into dimes and cents; but, in fact, it is not at all necessary to do so, because all the accounts of the United States are kept in dollars and cents.

	Ex. 3.			Ex. 4.			Ex. 5.	
	$	*cts.*		$	*cts.*		$	*cts.*
From	196	$62\frac{1}{2}$	From	750	75	From	100	16
Take	99	75	Take	189	$62\frac{1}{2}$	Take	99	$31\frac{3}{4}$
Rem.			Rem.			Rem.		

6. From 75 dollars take 75 cents. Ans. $74.25.

7. From 125 dollars, 55 cents, take 75 dollars, $62\frac{1}{2}$ cents. Ans. $$49.92\frac{1}{2}$.

8. From 100 dollars take $11\frac{1}{4}$ cents. Ans. $$99.88\frac{3}{4}$.

9. From 23 dollars take 1 dollar, 1 cent. Ans. $21.99.

10. Required the balance of this account:

Dr. $	*cts.*	James Shea.	Cr. $	*cts.*
756	19		8000	30
365	75		999	99
93	25		1430	33
14	19		6739	19
756	35		3000	75
1827	00		196	10
900	75			

Ans. Balance in favour of James Shea, $15653.18.

English Money:

Ex. 1. Required the difference between £159, 9*s.* $4\frac{1}{4}$*d.*, and £86, 17*s.* $8\frac{1}{2}$*d.*

£	*s.*	*d.*
159	9	$4\frac{1}{4}$
86	17	$8\frac{1}{2}$
72	11	$7\frac{3}{4}$

In this example, as a halfpenny is greater than a farthing, it is taken from a penny, and the remainder being added to the farthing, the sum, three farthings, is set down; a penny is then carried to 8 pence, and the sum being taken from one shilling, the remainder 3 is added to 4 pence, and the amount set down. We then proceed thus: 1 and 7 are 8; 8 from 9 and 1 remains; 1 from 2, (the tens' figure in the shillings in a pound,) and 1 remains; 1 and 6 are 7; 7 from 9 and 2 remain, &c.

Ex. 2. Suppose a person is debtor to sundry persons in the following sums:

	£	s.	d.
	7561	13	6¾
	1073	10	9
	3764	18	7½
	4567	19	11¼
	2000	11	11½
Dr.			

And is creditor, by book-debts, from different people in the following sums:

	£	s.	d.
	6343	12	6
	4000	19	9¼
	3009	16	10¾
	6703	14	11½
Dr.			
Cr.			

Ans. Balance in favour of Cr. £1089 9s. 3½d.

Troy Weight.

Ex. 1. From 17*lb.* 11*oz.* 19*dwts.* 21*gr.* take 8*lb.* 10*oz.* 19*dwts.* 22*gr.* Ans. 9*lb.* 19*dwts.* 23*gr.*

2. From 637*lb.* 9*oz.* 8*gr.* take 288*lb.* 1*oz.* 9*dwts.* 20*gr.* Ans. 349*lb.* 7*oz.* 10*dwts.* 12*gr.*

3 From 8947*lb.* take 5398*lb.* 6*oz.* 18*dwts.* 12*gr.* Ans. 3548*lb.* 5*oz.* 1*dwt.* 12*gr.*

Avoirdupois Weight.

Ex. 1.

	T.	*cwt.*	*qr.*	*lb.*	*oz.*
From	8	13	1	12	9
Take	3	12	2	25	11
Rem.	5	0	2	14	14

Ex. 2.

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
From	19	2	19	3	12
Take	12	2	18	2	15
Rem.	7	0	1	0	13

Ex. 3.

	T.	*cwt.*	*qr.*	*lb.*
From	17	19	3	17
Take	14	17	3	15
Rem.				

Ex. 4.

	lb.	*oz.*	*dr.*
From	12	12	12
Take	11	15	14
Rem.			

5. Bought 2 tons, 5*cwt.* 1*qr.* 7*lb.* of sugar, and sold 1 ton, 19*cwt*, 20*lb.* What remains? Ans. 6*cwt.* 15*lb.*

Apothecaries' Weight.

Ex. 1.

	℔	℥	ʒ	℈	*gr.*
From	12	9	3	1	12
Take	11	8	6	2	11
Rem.	1	0	4	2	1

Ex. 2.

	℔	℥	ʒ
From	96	11	7
Take	87	11	6
Rem.	9	0	1

Ex. 3. From 12℔, 9℥, 4ʒ, 14*gr.* take 9℔, 9℥, 6ʒ, 1℈, 15*gr.* Ans. 2℔, 11℥, 5ʒ, 1℈, 19*gr.*

4. From 96℔ take 95℔, 11℥, 7ʒ, 2℈, 19*gr.*
Ans. 1 grain.

Cloth Measure.

		yds.	*qrs.*	*n.*
Ex. 1.	From	176	1	1
	Take	99	3	3
	Rem.	76	1	2

		E. e.	*qr.*	*n.*
Ex. 2.	From	75	0	3
	Take	18	1	3
	Rem.	56	4	0

3. From 75*yds.* take 3*qrs.* 3*n.* Ans. 74*yds.* 0*qr.* 1*n.*

4. Bought a piece of blue cloth containing 49*yds.* and sold thereof 15*yds.* 1*qr.* 2*n.* What remains?
Ans. 33*yds.* 2*qrs.* 2*n.*

5. I cut 19*yds.* 2*qr.* from a piece of cloth containing 40*yds.* What remains? Ans. 20*yds.* 2*qrs.*

Long Measure.

Ex. 1.

	lea.	*m.*	*fur.*	*p.*	*yds.*
From	19	2	6	35	3
Take	16	1	7	39	4
Rem.	3	0	6	35	$4\frac{1}{2}$

Ex. 2.

	m.	*yds.*	*ft.*	*in.*
From	75	4	1	10
Take	37	4	2	11
Rem.				

3. Suppose the distance from New-York to Philadelphia to be 93 miles, 6 furlongs, 34 poles, and the distance from

New-York to Washington-city 254 miles: what is the distance from Philadelphia to Washington-city?

Ans. 160*m*. 4*fur*. 6*p*

4. From 75 leagues take 2 miles, 2 furlongs, 30 poles

Ans. 74*lea*. 5*fur*. 10*p*.

5. From 100 miles take 20 poles. Ans.

Square and Land Measure.

Ex. 1.

	yds.	*sq. ft.*	*sq. in.*
From	760	6	120
Take	625	8	124
Rem.	134	6	140

Ex. 2.

	p.	*yds.*	*sq. ft.*	*sq. in.*
From	76	20	3	143
Take	46	25	6	129
Rem.	29	24¼	6	14

Ex. 3.

	acr.	*R.*	*P.*
From	175	3	29
Take	126	3	39
Rem.	48	3	30

Ex. 4.

	acr.	*R.*	*P.*
From	200	0	0
Take	199	3	3
Rem.			

5. From 75 square yds. 6 square ft. 120 square inches, take 35 square yds. 7 square ft. 140 square inches.

Ans. 39 square yds. 7 square ft. 124 square in.

6. From 90 square yds. take 88 square yds. 8 square ft. 143 square inches. Ans.

7. A gentleman owned a farm containing 1000 acres; 400 acres of which he gave to his son, 312 acres, 2 roods, 20 poles to his daughter: how many acres had he left for himself? Ans. 287 acres, 1*R*. 20*p*.

Solid Measure.

Ex. 1.

	cds.	*ft.*	*cub. in.*
From	36	120	1176
Take	18	127	1296
Rem.	17	120	1608

Ex. 2.

	T.	*ft.*
From	76	30
Take	35	42
Rem.	40	38

3. From a pile of wood containing 300 cords I sell 120 cords, 119 cubic ft. and 120 cubic inches: what quantity remains? Ans. 179 cords, 8 cubic ft. 1608 cubic in.

Liquid Measure.

Ex. 1.

	t.	*hhd.*	*gal.*	*qt.*	*p.*
From	75	3	50	2	1
Take	13	3	50	3	1
Rem.	61	3	62	3	0

Ex. 2.

	gal.	*qt.*	*p.*	*g.*
From	39	2	1	1
Take	30	3	0	2
Rem.	8	3	0	3

3. Subtract 30 tuns, 1 pipe, 1 hhd. 30 gallons, 1 quart, 1 pint, 1 gill, from 31 tuns. Ans. 32*gal.* 2*qt.* 3*gills.*

4. Bought 2 pipes of Gin, containing 256 gallons; 39 gallons, 1 quart, 1 pint of which leaked. What remains?
Ans.

Dry Measure.

Ex. 1.

	b.	*p.*	*qt.*
From	30	2	2
Take	18	1	7
Rem.	12	0	3

Ex. 2.

	b.	*p.*	*qt.*
From	160	0	0
Take	159	3	7
Rem.			

3. From 439 bushels, take 200 bushels, 1 peck, 1 quart.
Ans. 238*b.* 2*p.* 7*qt.*

4. Bought 1000 bushels of corn, for $650.75; and sold 500 bushels, 3 pecks, 5 quarts of the same, for $375.25. How many bushels, &c. remain, and what does the remainder stand me in?

Time.

Ex. 1.

	dys.	*hrs.*	*min.*	*sec.*
From	352	16	16	30
Take	128	23	14	33
Rem.	223	17	1	57

Ex. 2.

	yrs.	*dys.*	*hrs.*
From	1827	125	12
Take	1828	335	16
Rem.			

3. How much time has elapsed from the 19th of April, 1823, till the 19th of March, in the present year, 1827.
Ans. 3*yrs.* 334*dys.*

Division of the Circle.

Ex. 1.

	°	′	″
From	35	29	33
Take	27	30	35
Rem.	7	58	58

Ex. 2.

	°	′	″
From	135	00	00
Take	78	15	20
Rem.			

3. The inclination of the orbit of Mercury to the plane of the ecliptic, is about 7°; and that of Venus, 3° 23′ 30″. Required their difference. Ans. 3° 36′ 30″.

4. The orbit of Mars is inclined to the plane of the ecliptic, 1° 51′ 7″; and that of Jupiter, 1° 18′ 47″. Required their difference. Ans. 32′ 20″.

Question.

When is compound subtraction used?
Repeat the rule of operation.

Exercises in Compound Subtraction.

Ex. 1. From three thousand dollars, take ninety-nine dollars, ninety-nine cents. Ans. $2900.01.

2. Suppose a merchant in New-York should send goods to his correspondent in London, to the amount of $4000, with orders for one gold watch at $125.75, another at $200 37½, and a third at $300, to be sent in return. How much would remain due to the New-York merchant?
Ans. $3373.87½.

3. Suppose the effects of a bankrupt amount to £500; and he owes to A 300 19*s.* 6*d.*; to B £519 7*s.* 6*d.*; to C £218 14*s.* 5*d.*; and to D £25 10*s.* What is the deficiency?
Ans. £564 11*s.* 5*d.*

4. A merchant commencing business with £10000, gains £1099 15*s.* 6*d.* in the course of a year; and at the expiration of that period, distributes in charity the sum of £114, 16*s.* 4*d.* What is the balance remaining on hand?
Ans. £10984, 19*s.* 2*d.*

5. A goldsmith had a wedge of gold weighing 12*lb.* 10*oz.* 12*dwts.* and 20*gr.*; he melts 9*lb.* 9*oz.* 12*dwts.* and 22*gr.* How much has he left. Ans. 3*lb.* 0*oz.* 19*dwts.* 22*gr.*

6. The great bell at Oxford weighs 7 *tons*, 11*cwt.* 3 *qr.* 4*lb.*; and that at St. Paul's, London, 5 *tons*, 2*cwt.* 1*qr.* 22*lb*: how much heavier than these together is the great bell at Moscow, which is 192 *tons*, 17*cwt.* 16*lb.*

Ans. 180 *tons*, 2*cwt.* 3*qr.* 18*lb.*

7. From 2 pounds, 2 ounces, 4 drams, 2 scruples, 15 grains, take 1 pound, 9 ounces, 5 drams, 2 scruples, 16 grains. Ans. 4℥, 6ʒ, 2℈, 19*gr.*

8. Bought 20 pieces of cloth, containing 656 yards; and sold 19 pieces, containing 620*yd.* 2*qr.* 2 *nails.* How many yards remain? Ans. 35*yd.* 1*qr.* 2 *nails.*

9. The distance from New-York to Boston is 210 miles; a person going from New-York to Boston, having travelled 82 miles, 3 furlongs, 30 poles. How many miles has he still to travel? Ans. 127*m.* 4*fur.* 10*p.*

10. From a field of $18\frac{1}{2}$ acres, I take out a garden, measuring 2 acres, 1 rood 30 poles; a piece of ground for coach-house, stables, &c. that measures 1 rood, 12 poles; and an orchard, measuring 5 acres, 2 roods, 10 poles. What will be the size of the field after these pieces are taken away? Ans. 10 *acres*, 28 *poles.*

11. My farm produced 150 bushels of wheat; 200 bushels of rye; and 350 bushels, 2 pecks of Indian corn: now admitting that I sold 100 bushels, 2 pecks, 2 quarts of each, how many bushels, &c. were left for the use of my family?

Ans. 398*b.* 3*p.* 2*qrt.*

12. The Articles of Confederation of the States, were adopted by Congress on the 15th of April, 1777; and on the 23d of February, 1813, the British brig Peacock was sunk by the American sloop of war Hornet. What time elapsed between these two events? Ans. 35 years, 314 days.

13. On the night of the 18th of December, 1774, the patriotic people of Boston emptied the contents of three hundred and forty two chests of tea into the ocean; and the Declaration of Independence was almost unanimously adopted on the 4th of July, 1776. What time elapsed between these two memorable events? Ans. 1 year, 199 days.

14. The inclination of the Moon's orbit to the plane of the ecliptic is about 5°, 9′; and the inclination of the orbit of Saturn to the plane of the ecliptic is 2° 30′ 18″. Required their difference. Ans. 2° 38′ 42″.

15. The latitude of Rome, (St. Peter's,) is 41° 53′ 54″, North; and of Paris, (Observatory of the Military School,)

48° 51′ 6″, North. Required the difference of latitude between these two places. Ans. 6° 57′ 12″.

16. The latitude of London, (St. Paul's,) is 51° 30′ 49″, North; and of Dublin, 53° 21′, North. Required their difference. Ans. 1° 50′ 11″.

17. The following are the times in which the primary planets perform their revolutions about the sun; required the differences of the first and second, of the second and third, &c.

	dys.	hrs.	min.	sec.
Mercury	87	23	15	44
Venus	224	16	49	11
The Earth	365	5	48	51
Mars	686	23	30	36
Vesta	1335	4	55	12
Juno	1590	23	57	7
Ceres	1681	12	56	10
Pallas	1681	17	00	58
Jupiter	4332	14	18	41
Saturn	10758	23	16	34
Uranus	30688	17	6	2

Answers.

Dif.	dys.	hrs.	min.	sec.
1st & 2d	136	17	33	27
2d & 3d	140	12	59	40
3d & 4th	321	17	41	45
4th & 5th	648	5	24	36
5th & 6th	255	19	1	55
6th & 7th	90	12	59	3
7th & 8th		4	4	48
8th & 9th	2650	21	17	43
9th & 10th	6426	8	57	53
10th & 11th	19929	17	49	28

Compound Multiplication.

52. When the multiplicand expresses a quantity of the same kind, but of more denominations than one, the process is termed *compound multiplication.*

Problem 1. *To multiply a number of more denominations than one, by a number not exceeding* 12.

Rule. Commencing with the lowest denomination, multiply successively the several numbers in the multiplicand by the multiplier, dividing, setting down, and carrying as in compound addition.

Ex. 1. Multiply 37 dollars, $37\frac{1}{2}$ cents by 6.

Here, $\frac{1}{2} \times 6$ is equal to 3, because 6 half-cents are equivalent to 3 cents; then 6 times 7 is 42 and 3 are 45, set down 5 and carry 4 as in multiplication of whole numbers.

$	cts.
37	$37\frac{1}{2}$
	6
225	25

2. Multiply £1 14*s.* 7¾*d.* by 9.

In this example the farthings, pence, and shillings are multiplied successively by 4, 12, 20, (or the tens of the shillings by 2;) the several remainders are written down, and the quotients carried. The pounds are multiplied as in simple multiplication; and the product is found to be £15, 11*s.* 9¾.

£	*s.*	*d.*
1	14	7¾
		9
15	11	9¾

3. Multiply 25 *tons*, 13*cwt.* 3*qr.* 14*lb.* by 8.

In this example, the *lbs. qrs.* and *cwts.* are multiplied successively by 28, 4, 20, (or the tens of the *cwt.* by 2,) the several remainders are written down, and the quotients carried.

tons.	*cwt.*	*qrs.*	*lb.*
25	13	3	14
			8
205	11	0	0

4. Multiply 72 pounds, 10 ounces, 19 pennyweights, 18 grains, by 5. Ans. 364*lb.* 6*oz.* 18*dwt.* 18*gr.*

5. Multiply $187.37½ by 10. Ans. $1873.75.

6. Multiply £175 14*s.* 10½*d.* by 8. Ans. £1405 19*s.* 0*d.*

7. Multiply 48 dollars, 37 cents, 6 mills, by 9. Ans. $435 38*cts.* 4*m.*

8. Multiply 36 pounds, 3 ounces, 3 drams, 1 scruple, 19 grains, by 2. Ans. 72℔ 6℥ 7ʒ 0℈ 18*gr.*

9. Multiply 75 yards, 2 quarters, 2 nails, by 3. Ans. 226*yd.* 3*qr.* 2 *nails.*

10. Multiply 75 miles, 3 furlongs, 30 poles, by 4. Ans. 301 *miles*, 7 *fur.*

11. Multiply 300 acres, 3 roods, 3 poles, by 5. Ans. 1503*a.* 3*r.* 15*p.*

12. Multiply 35 tuns, 1 hogshead, 36 gallons, 3 quarts, 1 pint, by 6. Ans. 212 *tuns* 1*hhd.* 32*gal.* 1*qt.*

13. Multiply 32 years, 325 days, 12 hours, by 7. Ans. 230*yrs.* 88*dys.* 12*hrs.*

14. Multiply 33° 12′ 13″ by 7. Ans. 232° 25′ 31″.

Problem 2.

To multiply by a number which exceeds 12, but is the product of two or more factors, each less than 13.

Rule.

53. By the preceding problem, multiply the given multiplicand, by one of the factors. Multiply the re-

sult by another. Multiply this last result by another, if there be so many; and thus proceed, whatever is their number.

Ex. 1. Multiply \$75.31¼, by 24.

In this example, the multiplicand is multiplied by 4, and the product is \$301.25. This again is multiplied by 6, and the product is \$1807.50. The reason of the operation is sufficiently obvious, since 24 is the product of 4 and 6. The work might be proved, by multiplying the multiplicand by 6, and the result by 4. When the multiplicand contains one or more quarters, if one of the factors be *even*, (that is, divisible by 2,) it is better to use it first, as the quarters may thus disappear, and the rest of the work be easier.

\$	*cts.*
75	31¼
	4
301	25
	6
1807	50

2. Multiply £756 13*s.* 9*d.* by 30.

In this example, the multiplicand is multiplied by 6, and the product is £4540 2*s.* 6*d.* This again is multiplied by 5, and the product is £22700 12*s.* 6*d.* The reason of the operation is sufficiently obvious, since 30 is the product of 6 and 5.

£	*s.*	*d.*
756	13	9
		6
4540	2	6
		5
22700	12	6

3. Multiply 11¾ cents by 16. Ans. \$1.88.
4. Multiply 12½ cents by 14. Ans. \$1.75.
5. Multiply \$1.37½ by 40. Ans. \$55.00.
6. Multiply 12 shillings and 6 pence by 36. Ans. £22 10*s.* 0*d.*
7. Multiply £1 1*s.* and 9*d.* by 48. Ans. £52 4*s.* 0*d.*
8. Multiply 3 pounds, 3 ounces, 19 pennyweights, 21 grains, by 42. Ans. 139*lb.* 11*oz.* 14*dwts.* 18*gr.*
9. Multiply 7*cwt.* 3*qr.* 14*lb.* by 60. Ans. 472*cwt.* 2*qrs.*
10. Multiply 3 tons, 3 hundred, 3 quarters, by 64. Ans. 204 *tons.*
11. Multiply 1 pound 1 ounce, 7 drams, 1 scruple, and 18 grains, by 70. Ans. 81℔ 4℥ 6ʒ 1℈.
12. Multiply 70 yards, 1 quarter, 1 nail, by 45. Ans. 3164*yds.* 0*qrs.* 1 *nail.*

13. Multiply 13 leagues, 2 miles, 3 furlongs, 30 poles, by 50. Ans. 691*lea.* 0*m.* 3*fur.* 20*p.*

14. Multiply 756 acres, 3 roods, 30 poles, by 108. Ans. 81749*a.* 1*r.*

15. Multiply 13 tuns, 2 hogsheads 31 gallons, by 75.* Ans. 1021 *tuns*, 2*hhd.* 57*gal.*

16. Multiply 30 bushels, 3 pecks, 4 quarts, by 112. Ans. 3458 *bush.*

17. Multiply 29 days, 12 hours, 30 minutes, 30 seconds, by 128. Ans. 10*yrs.* 128*dys.* 17*hrs.* 4 *min.*

18. Multiply 1° 30′ 30″ by 168. Ans. 253° 24′ 0″.

Problem 3.

To multiply by a number which exceeds 12, *but is not produced by factors below* 13.

Rule 1.

54. Use those factors whose product is nearly equal to the multiplier. Increase or diminish the result, as the case may require, by the product of the multiplicand, and the difference between the multiplier and the product of the factors employed.

Ex. 1. Multiply \$3 $62\frac{1}{2}$ by 38.

In this example, 38 not being the product of any two factors not exceeding 12, we multiply by 36, as before, and to the product we add twice the multiplicand, to find the product by 38. The answer would have been attained with nearly the same facility, had we multiplied by 40 (4×10,) and subtracted twice the multiplicand; and thus the operation might be proved.

\$	*cts.*	
3	$62\frac{1}{2}$	
	12	
43	50	product by 12
	3	
130	50	- - - 36
7	25	2
137	75	- - - 38

* In this and each of the following three examples, the multiplier is the product of *three* factors; $75=3\times5\times5$; $112=8\times2\times7$, or $4\times4\times7$; $128=8\times8\times2$, or $4\times4\times8$; and $168=8\times3\times7$, or $4\times6\times7$.

Rule 2.

55. Multiply the given price or quantity by 10, which will give the price of 12, this again multiplied gives the price of 100, this again by 10 for 1000, &c. Then multiply the first line or price of 1 by the units; the second product, or the price of 10, by the tens; and proceed in like manner for the hundreds, &c. The products then added together will be the answer required.

Ex. 2. What cost 2485 yards of broadcloth, at 15*s.* $7\frac{1}{2}$*d.* per yard?

£	*s.*	*d.*		
0	15	$7\frac{1}{2}$	= price of	1 yard.
		10		
7	16	3	= - - -	10
		10		
78	2	6	= - - -	100
		10		
781	5	0	= - - -	1000
		2		
1562	10	0	= - - -	2000
312	10	0	= - - -	400
62	10	0	= - - -	80
3	18	$1\frac{1}{2}$	= - - -	5
1941	8	$1\frac{1}{2}$	= - - -	2485

In this example, we find successively the prices of 10, 100, 1000. We then multiply the price of 1000 by 2; of 100 by 4; of 10 by 8; and of 1 by 5. We have thus the prices of 2000, of 400, of 80, and of 5, the sum of which is £1941 8*s.* $1\frac{1}{2}$*d.* the answer. This method is useful when the multiplier is very great, or when the factors, whose product is nearly equal to it, cannot be easily found.

3. Multiply \$1·$37\frac{1}{2}$, by 13. Ans. \$17·$87\frac{1}{2}$.

4. Multiply 75 cents by 23. Ans. \$17·25.

5. Multiply $6\frac{1}{4}$ cents by 29. Ans. \$1·81$\frac{1}{4}$.
6. Multiply 17*s*. $3\frac{1}{4}$*d*. by. 31. Ans £26 16*s*. $0\frac{1}{4}$*d*.
7. Multiply 11*s*. $3\frac{1}{4}$*d*. by 39. Ans. £22 1*s*. $2\frac{1}{4}$*d*.
8. Multiply 18*s*. 8*d*. by 46. Ans. £42 18*s*. 8*d*.
9. Multiply 7 pounds, 9 ounces, 3 pennyweights, 12 grains by 47. Ans. 364*lb*. 11*oz*. 4*dwt*. 12*gr*.
10. Multiply 30 tons, 10 hundredweight, 3 quarters by 52. Ans. 1587 *tons*, 19*cwt*. 0*qr*.
11. Multiply 31 yards, 2 quarters by 53. Ans. 1669*yd*. 2*qr*.
12. Multiply 30 miles, 3 furlongs, 29 poles by 58. Ans. 1767*m*. 0*fur*. 2*p*.
13. Multiply 36 square feet, 120 square inches by 65. Ans. 2394*sq.ft*. 24*sq.in*.
14. Multiply 37 acres, 1 rood, 10 poles by 68. Ans. 2537*a*. 1*r*.
15. Multiply 12 cubic feet, 1700 cubic inches by 69. Ans. 33*c.yd*. 4*c.ft*. 1524*c.in*.
16. Multiply 31 years, 302 days, 20 hours, 20 minutes by 76. Ans. 2419*yr*. 21*dy*, 9*hr*. 20*m*.
17. Multiply 13 gallons, 3 quarts, 1 pint, 2 gills by 27. Ans. 1*tun*, 1*hhd*. 61 *gal*. 1*qt*. 0*pt*. 2 *gills*.
18. Multiply 30 chaldrons, 2 bushels, 20 pecks, 2 quarts by 82. Ans. 2465*ch*. 30*b*. 0*p*. 4*qt*.
19. Multiply 3° 30′ 30″ by 89. Ans. 11 *signs* 25° 30′ 30″.
20. Multiply \$3·37$\frac{1}{2}$ by 150. Ans. \$506·25.
21. Multiply \$175·12$\frac{1}{2}$ by 155. Ans. \$27144·37$\frac{1}{2}$.
22. Multiply £2 11*s*. $5\frac{1}{4}$*d*. by 156. Ans. £401 4*s*. 3*d*.
23. Multiply $18\frac{3}{4}$ cents by 145. Ans. \$27·18$\frac{3}{4}$.
24. Multiply 6*s*. $9\frac{1}{2}$ by 139. Ans. £47 4*s*. $0\frac{1}{2}$*d*.
25. Multiply £2 7*s*. $8\frac{1}{2}$*d*. by 79. Ans. £188 8*s*. $11\frac{1}{2}$*d*.

Exercises in Compound Multiplication.

Ex. 1. Required the cost of a chest of tea, containing 97 pounds, at $87\frac{1}{2}$ cents per pound. Ans. \$84·87$\frac{1}{2}$.

2. Required the cost of a firkin of butter, weighing 72 pounds, at $18\frac{3}{4}$ cents per pound. Ans. \$13·50.

3. Required the cost of 25 pounds of beeswax, at $12\frac{1}{2}$ cents per pound. Ans \$3·12$\frac{1}{2}$.

4. Required the cost of 12 chaldrons of Liverpool coal, at \$12·50 per chaldron. Ans. \$150·00.

5. Required the cost of 35 chaldrons of Schuylkill coal, at $11·25 per chaldron. Ans. $393·75

6. Required the cost of 22 pounds of Cocoa, at $10\frac{1}{2}$ cents per pound. Ans. $2·31.

7. Required the cost of 120 pounds of coffee, Porto Rico, at $15\frac{1}{2}$ cents per pound. Ans. $18·60.

8. Required the cost of 119 pounds of coffee, St. Domingo, at $13\frac{3}{4}$ cents per pound. Ans. $16·36$\frac{1}{4}$.

9. Required the cost of 33 pounds of copper, at $27\frac{1}{2}$ cents per pound. Ans. $9·07$\frac{1}{2}$.

10. Required the cost of 256 pounds of cotton, New-Orleans, at $10\frac{1}{4}$ cents per pound. Ans. $26·24.

11. Required the cost of 25 yards of calico, at $18\frac{1}{2}$ cents per yard. Ans. $4·62$\frac{1}{2}$.

12. Required the cost of 36 yards of shirting, brown, at $8\frac{3}{4}$ cents per yard. Ans. $3·15.

13. Required the cost of 42 yards of check, at $13\frac{3}{4}$ cents per yard. Ans. $5·77$\frac{1}{2}$.

14. Required the cost of 29 pieces of diaper, Russia, at $2·25 per piece. Ans. $65·25.

15. Required the cost of 32 pounds of figs, at $8\frac{1}{2}$ cents per pound. Ans. $2·72.

16. Required the cost of 365 bushels of barley, at 95 cents per bushel. Ans. $346·75.

17. Required the cost of 26 gallons of honey, Havana, at 75 cents per gallon. Ans. $19·50.

18. Required the cost of 15 pounds of sugar, at $13\frac{1}{2}$ cents per pound. Ans. $2·02$\frac{1}{2}$.

19. Required the amount of a box of linen cloth containing as under:

pieces.		yd.	per yd. *s.*	*d.*	pieces.		yd.	per yd. *s.*	*d.*
2	containing	49 at	2	1	3	containing	75 at	3	3
2	- - - -	49	2	3	3	- - - -	75	3	5
2	- - - -	48	2	5	3	- - - -	75	3	7
3	- - - -	73	2	7	3	- - - -	75	3	9
3	- - - -	75	2	9	3	- - - -	68	3	11
3	- - - -	75	2	11	3	- - - -	75	4	2
3	- - - -	75	3	1					

Ans. £140 2*s.* 0*d.*

20. From 1783 to 1793, both inclusive, the money paid for slaves, imported into the West Indies, in Liverpool ves-

sels, was, at an average, £1380622 16s. 4¼d. each year. What was the entire amount? Ans. £15186851 0s. 1½d.

21. How much is the weight of 35 chests of tea, each weighing 96 pounds, 10 ounces? Ans. 3381lb. 14oz.

22. How much is the weight of 16 hogsheads of sugar, each weighing 7cwt. 3qr. 21lb. Ans. 127cwt. 0qr. 0lb.

23. How much is the weight of 28 ingots of gold, each weighing 6 pounds, 7 ounces, 15 pennyweights, 20 grains? Ans. 186lb. 2oz. 3dwt. 8gr.

24. What is the weight of 1000 dollars, each weighing 17 pennyweights 6 grains? Ans. 71lb. 10oz. 10dwt.

25. In 36 pieces of linen cloth, each measuring 25 yards, 2 quarters, 1 nail: how many yards? Ans. 920yd. 1qr.

26. How many acres are there in 34 farms, each containing 315 acres, 3 roods, 30 poles? Ans. 10741a. 3r. 20p.

27. In 17 pipes of brandy, each containing 127 gallons, 3 quarts, 1 pint, how many gallons? Ans. 2173gal. 3qt. 1pt.

28. The Moon performs her *mean sidereal revolution* in 27 days, 7 hours, 43 minutes, and 11½ seconds. In how many days will she perform 13 revolutions? Ans. 355 dys. 4hrs. 21′ 29½″.

29. The *mean daily motion* of the planet Venus in her orbit, is 1 degree, 36 minutes, 8 seconds, per day. How many degrees, &c. does she describe in 224 days? Ans. 358° 53′ 52″.

30. The *mean daily motion* of the Earth in its orbit, is 59 minutes, 8 seconds. How many degrees will the Earth describe in 365 days? Ans. 359° 43′ 40″.

31. *Bills of Parcels.*

New-York, March 27th, 1817.

Mrs. Duff,

Bought of John Murray,

		s.	d.		£	s.	d.
9	yards of silk, at	13	6	per yd.			
14½	yards of flowered do. at	15	11	per yd.			
23	yards of sarsanet, at	6	4	per yd.			
13	yards of brocade, at	9	11½	per yd.			
15¼	yards of velvet, at	19	10	per yd.			
18	yards of lustring at	5	8	per yd.			
					£51	11	10¼

82. *A Milliner's Bill.*

New-York, March 27th, 1827.

Mrs. Phillips,

Bought of Margaret Jones,

12 yards of fine lace, at	\$2.12½	per yd.
15 pair of kid gloves, at	\$0.87½	per pair.
19 French mounted fans, at	\$1.12½	each.
7 fine tippets, at	\$1.37½	each.
4 pair of silk gloves, at	\$0.44	per pair.
8 sets of knots, at	\$0.56¼	per set.
3 hats, at	\$9.75	each.
		\$105.13½

Questions.

When is compound multiplication used?

When the multiplier does not exceed 12, how is the operation performed?

When the multiplier exceeds 12, but is the product of two or more known factors, each less than 13, how is the operation performed?

Repeat the rules of operation, when the multiplier is not produced by factors below 13.*

* Compound Multiplication is seldom employed except in relation to money; but if it be necessary to use it in cases not illustrated here, no difficulty can arise, as the method is similar in all cases.

It may perhaps be proper to caution learners against the absurdity of attempting *to multiply money by money.* This caution will *not* appear unnecessary, if it be considered, that whole pages have been filled with instructions how to perform this problem; and it has been attempted to be shown, even with the semblance of *geometrical demonstration*, that if 2*s*. 6*d*. be multiplied by 2*s*. 6*d*. the product will be 3¼*l*. or 6*s*. 3*d*. Let it be considered, however, that in multiplication a quantity is simply *repeated* a given number of times: thus, if 2*s*. 6*d*. be repeated 4 times, the amount is 10*s*.; if 5 times, 12*s*. 6*d*. To talk, therefore, of multiplying 2*s*. 6*d*., by 2*s*. 6*d*, or, which amounts to the same thing, *repeating 2s. 6d., 2s. 6d. times*, is absolute nonsense. In the rule of proportion, indeed, we sometimes *appear* to multiply such quantities. Thus, in finding the interest of a sum at a given rate, for a year, we multiply by the rate and divide by 100. In this case, however, both 100 and the rate are divested of their characters as expressions for money, and are merely to be regarded as abstract numbers, used as the terms of a ratio. By multiplying by the rate, suppose 5, we merely repeat the principal

Compound Division.

56. When the dividend expresses a *quantity of the same kind, but of different* denominations, the process is termed *Compound Division.*

Problem 1. *To divide a number of more denominations than one, by a number not exceeding* 12.

57. Rule. Divide the highest denomination by the given divisor by short division. Reduce the remainder, if there be any, to the denomination next lower, and add to the result what was given of that denomination. Divide the sum by the divisor; and thus proceed to the lowest denomination or till nothing remains.

Ex. 1. Divide \$ 17·35 by 2.

Here the division is performed as in whole numbers; but it may be observed that the mark of dollars being prefixed, the cents must be separated from the dollars by a point.

\$	*cts.*
2)17	35
\$ 8·	$67\frac{1}{2}$

2. Divide £ 14 16*s.* $7\frac{1}{4}$*d.* by 10.

In this example, after dividing £ 14 by 10, we have £ 4, or 80 shillings; which, increased by 16, becomes 96*s.* Hence we find the next part of the quotient to be 9*s.* and the remainder is 6*s.* or 72*d.* which, increased by 7, becomes 79. This being divided by 10, we have the remainder 9*d.* or 36 far-

£	*s.*	*d.*	
10)14	16	$7\frac{1}{4}$	
1	9	$7\frac{3}{4}$	. . . 7*far.* or $1\frac{3}{4}$*d.*
		10	
14	16	$7\frac{1}{4}$	proof.

5 times, or find a principal 5 times as great; and then, as there must be one pound of interest for each hundred pounds in this increased principal, we try by division how often it contains one hundred pounds; and we thus find the pounds of the interest. In like manner, the multiplication of 200 cents by 200 cents, is *complete nonsense,* except there be a given term of comparison; for instance, one dollar or one cent; that is, if one dollar gain two hundred cents or two dollars, two dollars or two hundred cents would gain four dollars. But, if one cent would gain 200 cents, then 200 cents would gain 40000 cents, or 400 dollars.

things, to which the odd farthing is annexed; and continuing the division, we find the entire quotient to be £ 1 9s. 7¾d. and the remainder 7 farthings, or 1¾d. The proof is performed as in Division of Whole Numbers.

3. Divide $ 3·84 by 4. Ans. 96 cents.
4. Divide $ 456·06 by 5. Ans. $91·21¼.
6. Divide $ 1·87½ by 3. Ans. 62½ cents.
7. Divide $ 375·75 by 5. Ans. $ 75·15.
8. Divide £ 75 13s. 6d. by 7. Ans. £ 10 16s. 2¼d.
10. Divide 3*lb.* 11*oz.* 3*dwt.* 19*gr.* by 8.
Ans. 5*oz.* 17*dwt.* 23⅞*gr.*
11. Divide 12 tons, 13*cwt.* 3*qrs.* 18*lb.* by 9.
Ans. 1*T.* 8*cwt.* 0*qr.* 23*lb.* 12*oz.* 7⅑*dr.*
12. Divide 37*yds.* 3 *qrs.* 2*n.* by 10.
Ans. 3*yds.* 3*qrs.* 0⅗ nails.
13. Divide 37 acres, 3 roods, 6 poles by 11.
Ans 3 acres, 1 rood, 29$\frac{7}{11}$ poles.
14. Divide 75 tuns, 1*hhd.* 33*gals.* by 12.
Ans. 6 tuns, 1*hhd.* 8*gals.*
15. Divide 30° 15′ 30″ by 5. Ans. 6° 3′ 6″.
16. Divide 63 days, 5 hours 30 minutes, 30 seconds, by 6.
Ans. 10 days, 12 hours, 55 minutes, 5 seconds.

PROBLEM 2. *To divide by a number which is greater than 12, but is the product of two or more factors, each less than 13.*

58. RULE. Divide the given number, by short division, by one of the factors. Divide the quotient by another factor. Divide the quotient thus obtained by another, if there be so many, and thus proceed, whatever may be their number.

Ex. 1. Divide $750·35 by 16.

In this example the factors are 4 and 4. In the division by 4, in the first place, the quotient is $187·58, and the remainder 3*cts.*; and again, in the division of this quotient by 4, the quotient resulting is $46·89½; or the quotient resulting is $46·89, and the remainder 2. This remainder being multiplied by 4, the first divisor, and the product increased by the former remainder 3, (see page 43) the true remainder is found to be 11 cents; so that the true quotient is $46·89 11/4. In the proof by multiplication, the remainder must be always added to the final product.

$	cts.	
4)750	35	
4)187	58	 3
$46	89½	
	4	
187	58	
	4	
750	32	add
	3	
$750	35	

2. Divide £59 13*s.* 3½*d.* by 66.

£	s.	d.	
6)59	13	3½	
11) 9	18	10½	 2
0	18	0¾	. 56*far.* or 1*s.* 2*d.*
		6	
5	8	4½	
		11	
59	12	1½	add
	1	2	
59	13	3½	proof.

In this example the factors are 6 and 11. In the division by 6, the quotient is £9 18*s.* 10½*d.* and the remainder 2 farthings; and in the division of this quotient by 11, the quotient resulting is 18*s.* 0¾*d.* and the remainder 9. This remainder being multiplied by 6, the first divisor, and the product increased by the former remainder, 2, (see p. 32.) the true remainder is found to be 56 farthings, or 1*s.* 2*d.* In the proof by multiplication, this remainder must be added to the final product.

3. Divide \$275 by 24. Ans. \$11·45·8⅓.
4. Divide \$182·37½ by 25. Ans. \$7·29·5.
5. Divide £58 7*s.* by 36. Ans. £1 12*s.* 5*d.*
6. Divide 17 tons, 10*cwt.* 3*qrs.* 27*lb.* by 28.
Ans. 12*cwt.* 2*qrs.* 3*lb.* 15*oz.* $6\frac{6}{7}$*dr.*
7. Divide 3*lb.* 10*oz.* 12*dwts.* 15*gr.* by 48.
Ans. 19*dwts.* $10\frac{3}{16}$*gr.*
8. Divide 75*yds.* 3*qrs.* by 64.
Ans. 1*yd.* 0*qr.* 2 nail, $2\frac{7}{64}$ inches.
9. Divide 320 acres, 2 roods, by 27.
Ans. 11acres. 3*r.* $19\frac{7}{27}$*p.*
10. Divide 365 days, 5*hrs.* 48*min.* 48*sec.* by 49.
Ans. 7 days, 10*hrs.* 53*min.* $38\frac{44}{49}$*sec.*
11. Divide 180° by 360. Ans. 30′.

Problem 3.—*To divide by a number which is greater than 12, and is not produced by factors below 13.*

59. **Rule.** The process is to be conducted as in Problem 1, except that Long Division is to be employed, instead of Short.

Ex. 1. Divide £2074 6*s.* 9¼*d.* by 597.

£	*s.*	*d.*	£	*s.*	*d.*
597)2074	6	9¼ (3	9	5¼ Ans.	
1791				6	
283		20	16	10½	
20				10	
5666		208	8	9	
5373				10	
293		2084	7	6	
12		10	8	5¼	
3525		2073	19	0¾	
2985			7	8½	
540		£2074	6*s.*	9¼*d.*	
4					
2161					
1791					
370 farthings, or 7*s.* 8½*d.*					

Here the first part of the quotient is £3, and the remainder £283 6*s*. or 5666 shillings, from which, as in division of whole numbers, we obtain 9*s*. and a remainder of 293*s*. 9*d*. or 3525*d*. Dividing this by the divisor, we obtain for the quotient 5*d*. and a remainder of 540¼*d*. or 2161 farthings, which gives a quotient of 3 farthings, and a remainder of 370 farthings, or 7*s*. 8½*d*. The work is proved as in the example.

2. Divide \$756·75 by 13. Ans. \$44·36·5$\frac{5}{13}$.

3. Divide £50 4*s*. 2*d*. by 19.
Ans. £2 12*s*. 10*d*.—*rem*. 4*d*.

4. Divide £115 12*s*. 6*d*. by 37. Ans. £3 2*s*. 6*d*.

5. Divide 16 tons, 15*cwt*. 3*qrs*. 12*lb*. by 26.
Ans. 12*cwt*. 3*qrs*. 18*lb*. 12*oz*. 4$\frac{12}{13}$*dr*.

6. Divide 1756 acres, 3 roods, 12 poles, by 103.
Ans. 17 acres 0 roods, 9$\frac{5}{103}$ poles.

7. Divide 19 tuns, 1*hhd*. 30*gals*. of wine by 365.
Ans. 13*gals*. 1*qt*. 0*pts*. 3$\frac{337}{365}$ gills.

8. Divide 435*yds*. by 78. Ans. 5*yds*. 2*qrs*. 1$\frac{3}{13}$*n*.

9. Divide 376 bushels by 37. Ans. 10*bush*. 0*p*. 5$\frac{3}{37}$*qt*.

10. Divide 17 leagues, 1 mile, 7 furlongs, by 201.
Ans. 2*fur*. 4 poles, 0*yds*. 2*ft*. 11 inches 5$\frac{111}{201}$ lines.

Exercises in Compound Division.

Ex. 1. If the duty on a pipe of Port wine, containing 138 gallons, be £52 6*s*. 6*d*. how much is the duty on each gallon? Ans. 7*s*. 7*d*.

2. If a chest of tea, containing 96 pounds, cost \$98·75, what cost 1 pound? Ans. \$1·02·8$\frac{31}{48}$.

3. If a contribution of \$27000 is to be made up in equal shares by 625 persons, how much must each contribute? Ans. \$43·20.

4. If a person spend \$1125 a-year, how much does he spend, at an average, each day? Ans. \$3·08·2$\frac{70}{365}$.

5. The prize-money to the amount of \$2575·75 is to be divided equally among 70 seamen, how much will each receive? Ans. \$36·79·6$\frac{3}{7}$.

6. The prize in a lottery is \$15000, what is the sixteenth share? Ans. \$937·50.

7. The following are the amounts of duties paid at the custom house of Belfast, Ireland, during the undermentioned years; required the annual sum at an average.

	£		£		£
In 1800,	62668	In 1806,	207382	In 1812,	395254
1801,	182314	1807,	320981	1813,	450498
1802,	270434	1808,	318121	1814,	373721
1803,	201180	1809,	425174	1820,	306263
1804,	207402	1810,	321325	1821,	386709
1805,	228645	1811,	344449		

Ans. £294265 17*s*. $7\frac{13}{17}$.

8. In 1821, the population of Great Britain and Ireland was 21238580, and the net amount of the public revenue of the United Kingdom was £58108855 2*s*. 2¾*d*. What was the quota of this amount paid by each individual at an average? Ans. £2 14*s*. 8½*d*. Rem. £12489 8*s*. 0¾*d*.

9. How much land is there, at an average, for each individual in England, which contains 50535 square miles, the population, according to the census of 1821, being 11261437.

Ans. 2 *acres*, 3 *roods*, $19\frac{5784417}{11261437}$ *poles*.

10. How much land is there, at an average, for each individual in Ireland, which contains 12001200 acres, supposing the population in 1821 to be 6846949?

Ans. 1 *acre*, 3 *rds*. $0\frac{3046280}{6846949}$ *p*. or 1 *acre*, 3 *rds*. nearly.

Questions.

When is compound division used?

How is the operation performed, when the divisor does not exceed 12?

How is the operation performed, when the divisor exceeds 12, but is the product of two or more known factors?

When the divisor exceeds 12, and is not produced by factors below 13?

CHAPTER IV.

On the Doctrine of Ratio and Proportion.

60. It may not be here improper to remark, that when any number is multiplied by another, the product is called a *multiple* of the multiplicand; and the multiplicand is called a *submultiple* of the product: and any number multiplied by itself is called the *square* of that number, or its *second power*.

Thus, 54 is a multiple of 6, and 6 a submultiple of 54; because 54 is equal to 9 times 6. Thus, again, 2 or 3 or 6 or 9 is a submultiple of 18. Submultiples are otherwise called *aliquot parts*. And again, the product 8×8 or 64 is the square of 8; and 8 is called the *square* root of 64.

61. Now, when two numbers are multiplied each by the same number, the products are called *equimultiples* of the respective multiplicands; and the latter are called *equisubmultiples* of the products.

Thus, 18 and 24 are equimultiples of 3 and 4, or 3 and 4 equisubmultiples of 18 and 24: because 18 is 6 times 3, and 24 is 6 times 4.

62. By the ratio of two quantities we mean their relative magnitudes, or the magnitude of one in comparison of the other.

Thus, although the absolute magnitude of a mile and 12 miles, is much greater than that of an inch and a foot, yet the relative magnitude, or ratio of the former two, is just the same with that of the latter; or in other words, a mile is just as small a space in comparison of 12 miles, as an inch is in comparison of a foot.

63. A ratio is written by the aid of two dots interposed between the terms of the ratio, of which the former is called the *antecedent*, and the latter, the *consequent*

And the ratio is called a ratio of *greater* or of *less inequality*, according as the antecedent is greater or less than the consequent.

Thus, 3 : 5 expresses the ratio of 3 to 5; in which 3 is the antecedent, and 5 the consequent; and the ratio is a ratio of less inequality. But the ratio of 7 to 5 is a ratio of greater inequality. The ratio of 5 to 7 is called the *reciprocal*, or *inverse* of the ratio of 7 to 5.

64. The ratio of any two numbers is the same with the ratio of any equimultiples, or equisubmultiples of those numbers.

This is an important principle of very extensive application: and its truth will appear most manifest on a little consideration. Thus, if we take the ratio of 3 to 5, and multiply both terms of it by 7: the products 21 and 35 are equimultiples of 3 and 5; and the ratio of 3 to 5 must be the same with the ratio of those products, because it is evidently the same with the ratio of 3 times 7 to 5 times 7. Or, to take another instance, is it not evident, that the ratio of 9 to 6 is the same with the ratio 900 to 600, or 90 to 60, that is, 9 tens to 6 tens, or in short of 9 times any number to 6 times the same number; that is, the same with the ratio of any equimultiples of 9 and 6? And is it not equally evident, that the ratio of 9 to 6 is the same with the ratio of the third part of 9 to the third part of 6; that is, 3 to 2, or of any other equisubmultiples of 9 and 6? This indeed, if it were needful, might be deduced by necessary inference from the former; inasmuch as 9 and 6 are equimultiples of 3 and 2, or any equisubmultiples of 9 and 6; and therefore in the same ratio with them.

65. The equality or identity of two ratios is denoted by four dots, (or sign of equality =,) interposed between the ratios.

Thus, 9 : 6 : : 3 : 2, or 9 : 6=3 : 2, denotes that the ratio of 9 to 6 is the same with, or equal to the ratio of 3 to 2; or as we commonly more briefly express it, that 9 is to 6 as 3 to 2; such a series is called a series of *proportionals*, or by one word, borrowed from the Greek language, an *analogy*. The first and fourth terms of such a series, (that is, the antecedent of the first ratio and the consequent of the second,) are called the *extremes*: the second and

third terms, (that is, the consequent of the first ratio, and the antecedent of the second,) are called the *means*. If the antecedent of the second ratio be the same with the consequent of the first, the terms are said to be in *continued proportion*. Thus, the numbers 3, 9, and 27 are in continued proportion; because, 3 : 9 : : 9 : 27.

66. If any two ratios be equal, it is plain that their *reciprocals* must be equal; that is, that the consequent of the first ratio is to its antecedent as the consequent of the second ratio to its antecedent.

Thus, since 9 : 6 : : 3 : 2, we may infer, that 6 : 9 : : 2 : 3. For if 9 be as much greater in comparison of 6, as 3 is in comparison of 2, it follows, that 6 is as much less in comparison of 9, as 2 is in comparison of 3.

67. Again, from any analogy we may infer, that the first antecedent is to the second antecedent as the first consequent is to the second consequent.

Thus, since 9 : 6 : : 3 : 2, we may infer, that 9 : 3 : : 6 : 2. For the two given ratios could not be equal, unless 9 were just as much greater in comparison of 3, as 6 is in comparison of 2. And from any given analogy we may also infer, that any equimultiples, or equisubmultiples of the antecedents, bear the same ratio to their respective consequents; and that the antecedents bear the same ratio to any equimultiples or equisubmultiples of their consequents. Thus, since 9 : 6 : : 3 : 2, we may infer, that 5 times 9 is to 6 as 5 times 3 to 2; or, that the fifth part of 9 is to 6 as the fifth part of 3 to 2. For it is plain that if we increase or diminish the corresponding terms of equal ratios *proportionally*, the resulting ratios must still be equal.

68. If we have given the first three terms of an analogy, we may find the fourth, by taking the product of the second and third terms, and dividing that product by the first.

Thus, suppose we want to find a fourth proportional to the numbers 3, 4 and 6; that is, such a number that the ratio of 3 to 4 shall be the same with the ratio of 6 to the fourth number found. Multiply 6 and 4, and divide their product,

24, by 3: the quotient 8 is the fourth proportional sought. The truth of this result is evident in the present instance; 6 the antecedent of the second ratio being twice 3 the antecedent of the first; and therefore the ratio of 3 to 4 must be the same with the ratio of twice 3 to twice 4; that is, of 6 to 8. But suppose the three given terms are 3, 4 and 5, the fourth proportional is found by the same process: divide 20, the product of the given means, by 3, the first term, the quotient 6 and $\frac{2}{3}$ (or 6 and the third part of 2) is the fourth term sought; which may be demonstrated thus: by the principles laid down in § 64, the ratio of 3 to 4, is the same with the ratio of their equimultiples, 5 times 3 to 5 times 4: or, again, is the same with the ratio of the equisubmultiples of the latter, the third part of 5 times 3 to the third part of 5 times 4; but the third part of 5 times 3 is 5; therefore, 3 is to 4 as 5 to the third part of 5 times 4, that is, to the quotient arising from dividing the product of the given means by the first term. Although the preceding demonstration involve no principle, but what must be sufficiently evident to a considerate mind; yet it may be satisfactory to some, that another demonstration of the same thing should be exhibited. Let us then again suppose that we want to investigate a method for finding a fourth proportional to 3, 4, and 5: we know that 3 is to 4 as 1 (the third part of 3) to the third part of 4; or as the equimultiples of the latter terms, 5 times 1, that is, 5 to 5 times the third part of 4. Thus we arrive at the same result as before; for 5 times the third part of 4; and the third part of 5 times 4, are equivalent, as the former must be three times less than 5 times 4, and therefore equal to its third part. This will be more fully shown when we come to treat of the doctrine of fractions.

69. In any analogy the product of the extremes is equal to the product of the means.

This immediately follows from what has been last demonstrated: since either extreme is equal to the product of the means divided by the other extreme; for instance, 5 : 7 : : 10 : 14. But we have seen that 14 is equal to the quotient arising from dividing the product of 7 and 10 by 5; therefore multiplying 14 by 5, must give a product equal to the product of 7 and 10.

70. We may also infer that, if two products be equal.

their factors are *reciprocally proportional*; that is, that the multiplier of one is to the multiplier of the other, as the multiplicand of the latter to the multiplicand of the former.

Thus, the product of 2 and 28, is equal to the product of 7 and 8; whence we may infer that 2 : 7 : : 8 : 28.

71. In any multiplication, unity is to either factor as the other factor to the product.

Thus, the product of 6 and 5 is 30; and 1 : 6 : : 5 : 30. This immediately appears, either from the last section, or from § 64, inasmuch as 5 and 30 are equimultiples of 1 and 6, and therefore in the same ratio.

72. In any division, the divisor is to unity as the dividend to the quotient.

Thus, dividing 36 by 4 the quotient is 9; and 4 : 1 : : 36 : 9. This appears from § 70, and from the principle that the dividend is always equal to the product of the divisor and quotient.

73. When we say that one quantity is *directly* as another quantity, it is to be understood that the one increases or diminishes in the same ratio in which the other increases or diminishes.

For example, if I purchase cloth at 7 dollars a yard, the amount of the cost depends upon the quantity purchased, and is therefore said to be directly as that quantity.

74. But when one quantity increases in the same ratio in which another diminishes, or diminishes in the same ratio in which the other increases, the one is said to be *inversely* as the other.

For instance, if I have to ride a certain distance, the time requisite depends upon the speed employed, and is therefore said to be inversely as that speed.

75. In multiplication, the product is directly as either factor when the other is given.

Thus, if we multiply 7 first by 3 and then by 5, the product 21 and 35 are as 3 to 5.

76. But in division, the quotient is directly as the

dividend when the divisor is given; and inversely as the divisor when the dividend is given.

Thus, if we divide 24 and 27 by 3, the quotients 8 and 9 are in the ratio of 24 to 27; that is, 8 : 9 : : 24 : 27. But if we divide 24 first by 3 and then by 6, the quotients 8 and 4 are in the ratio of 6 to 3; that is, 8 : 4 : : 6 : 3.

77. Hence, whenever any quantity so depends upon two others, that it is directly as each of them when the other is given, it must vary in the ratio of the product of two numbers taken proportional to those two quantities.

Thus, the distance to which a man rides depends upon the time for which he rides, and the speed at which he rides; so as to be directly as either of them when the other is unvaried. If, therefore, A rides for 3 hours and B for 5 hours, and A ride twice as fast as B, the number of miles which A rides must be to the number of miles which B rides as 6 : 5, the products of the numbers which are proportional to their times and speed.

78. But whenever any other quantity so depends upon two others, that it is directly as the first when the second is given, and inversely as the second when the first is given, it must vary as the quotient obtained by dividing the first by the second; that is, dividing numbers proportional to these quantities.

Now, if I ride a journey, the requisite time so depends on the distance which I have to ride and the speed which I employ. It is directly as the distance, and inversely as the speed. If, therefore, A has to ride 50 miles and B 40, and A ride twice as fast as B, the time in which A performs his journey must be to the time in which B performs his as $\frac{50}{2}$ to $\frac{40}{1}$; that is, 25 to 40, or 5 to 8.

79. Any two products are said to be to each other in a ratio *compounded* of the ratios of their factors.

Thus, the ratio compounded of the ratios of 2 : 5 and 7 : 3 is the ratio of 14 : 15. Hence, the ratio compounded of two equal ratios is, the ratio of the squares of the terms

of either ratio. Thus, the ratio compounded of the equal ratio 9 : 6 and 3 : 2 is the ratio of 81 : 36, (9×9 : 6×6) or 9 : 4 (3×3 : 2×2.) For since 9 : 6 : : 3 : 2, it follows, § 67, that multiplying both antecedents by 3 and both consequents by 2, 27 : 12 : : 9 : 4 ; or multiplying both antecedents by 9 and both consequents by 6, that 81 : 36 : : 27 : 12. But the ratio 27 : 12 is, by definition, the ratio compounded of the ratios 9 : 6 and 3 : 2. And thus it appears that, if any four numbers be proportional, their squares are proportional.

Hence also it is evident that the ratio compounded of any ratio and its reciprocal, is a ratio of equality. Thus the ratio compounded of the ratios of 9 : 6 and 6 : 9 is the ratio of 54 : 54 ; that is, a ratio of equality.

80. Again, any ratio being given, we may conceive any number whatsoever interposed between its terms, and the given ratio as compounded of the ratios of the antecedent to the interposed number, and of the interposed number to the consequent.

Thus, the ratio of 9 to 6 may be considered as compounded of the ratios of 9 : 2 and 2 : 6. For 9 is to 6 as twice 9 to twice 6, which is the compounded ratio mentioned.

81. From what has been said it is manifest, that the problem of finding a fourth proportional to three given numbers will frequently admit of an abbreviated solution, by substituting lower numbers.

For in the first place, if the first two terms, or terms of the given ratio, admit of being divided evenly by the same number, we may substitute for them the resulting quotients, as being in the same ratio. Thus, if it be required to find a fourth proportional to 27, 63, and 21, solving the problem at large by rules laid down in § 68, we should have to take the product of 63 and 21, and then divide that product 1323 by 27, which gives the quotient 43 as the fourth proportional required. But 3 and 7 being equisubmultiples of 27, and 63 are in the same ratio ; and operating with these lower numbers we find the same result. It may be proved, in like manner, that, whenever the first and third terms admit of being evenly divided by the same numbers, we may

substitute the resulting quotients; for those equisubmultiples of the given antecedents must be proportional to the given consequent and the consequent sought.

82. Let it be required to find a number, to which a given number shall be in a ratio compounded of two or more given ratios.

The ratio compounded of two given ratios is (by definition) the ratio of the products of their respective terms. Therefore this problem resolves itself into that of finding a fourth proportion to three given terms.

Thus, if we want to find a number to which 6 shall be in a ratio compounded of $9 : 5$ and $15 : 36$, it is the same thing as if we were required to find a number to which 6 shall be in the ratio $9 \times 15 : 5 \times 36$. But it is plain that both terms of this ratio are divisible by 9 and by 5, and that we may therefore substitute the ratio of the resulting quotients, $3 : 4$, so that the number sought is $6 \times 4 \div 3$, or 8.

Hence it appears that, in solving this problem, if the antecedent and consequent of either the same or different ratios admit of being evenly divided by the same number, we may substitute the resulting quotients; and that we therefore ought not to take the products of the corresponding terms of the ratios which we want to compound, till we have inspected them for the purpose of ascertaining whether they are capable of being thus reduced; nor till we have compared the antecedents of the given ratios with the given antecedent of the ratio whose consequent we seek. For in the last instance, after reducing the question to that of finding a fourth proportional to 3, 4, and 6, the terms may be reduced still lower by substituting for 3 and 6 their equisubmultiples 1 and 2. And thus a question, at first involving very high numbers and appearing to require a very tedious operation, may frequently admit of a very brief solution.

83. The Rule, § 68, for finding a fourth proportional is commonly called "the Rule of Three," because we have three terms of an analogy given to find a fourth. It may more justly be called the *Rule of Proportion.*

Its very extensive practical application will be shown in Chapter VII.: meanwhile the young student may exercise

fraction is that whose numerator is less than its denominator. If the numerator be equal to the denominator, or greater, the fraction is called *improper*.

85. The denominator always denotes the number of equal parts, into which the whole thing or integer, is conceived to be divided. The numerator denotes the number of those parts, which are taken in the fraction.

Thus, the fraction $\frac{3}{7}$ intimates that the integer is divided into 7 equal parts, and that we take 3 of those parts in the fraction. Hence any improper fraction whose numerator and denominator are equal, such as $\frac{7}{7}$, $\frac{4}{4}$, &c. is equivalent to the one integer which we suppose divided into equal parts: for if we divide a pound, for instance, into 7 equal parts, and take 7 of those parts, we just take the whole pound, neither more nor less. On the other hand it is manifest that $\frac{6}{7}$, or any proper fraction, is less than the whole. Observe, that we consider and speak of the whole thing divided as *one* integer, whether it consist of a single pound, foot, yard, &c. or of ever so many feet, yards, &c.

According to the view which has hitherto been given of any fraction, such as $\frac{2}{3}$, we consider it as two-thirds of one. But there is another view also which it will be useful to attend to. It may be considered as the third part of two. This view arises immediately out of the former; for inasmuch as the third part of two is twice as great as the third part of one, it must be just equal to two-thirds of one. In like manner, the fraction $\frac{3}{7}$ may be indifferently considered either as three-sevenths of one, or as the seventh part of three; the latter being three times greater than the seventh part of one, and therefore just equal to three-sevenths of one. Thus any fraction may be considered as a quotient arising from the division of the numerator by the denominator. And hence the fractional notation is commonly employed to express division.

86. The value of any fraction varies directly as the numerator, and inversely as the denominator.

This appears at once from what has been last said, compared with § 73 and 74. The same thing also will appear from the first view given of a fraction, when we consider

that if a whole thing be divided into a given number of equal parts, the greater the number we take of those parts, the greater the quantity we take and in the same ratio: but the greater the number of equal parts into which the whole thing is divided, the less is any one of them, or any given number. Thus $\frac{3}{4}$ is greater than $\frac{3}{7}$ in the ratio of 7 : 4. ;—but $\frac{3}{7}$ is less than $\frac{5}{7}$ in the ratio of 3 : 5. Therefore $\frac{3}{4}$ is to $\frac{5}{7}$ in a ratio compounded of 3 : 5 and 7 : 4, (the direct ratio of the numerators and inverse ratio of the denominators,) that is, as 21 : 20.

87. Any fraction is to 1 as the numerator of the fraction to its denominator.

Thus $\frac{3}{7}$ is to 1 as 3 : 7. For 1 is equal to $\frac{7}{7}$. But $\frac{3}{7}$ is to $\frac{7}{7}$ as 3 to 7. Here and throughout the whole subject, when we speak of 1, it is to be understood in the sense explained at the end of § 85.

88. The value of any fraction will remain unaltered if we multiply or divide both its terms by the same number; that value depending altogether on the ratio of its terms, and not their absolute magnitude.

Thus the fraction $\frac{3}{4}$ is equal to the fraction $\frac{6}{8}$ or $\frac{9}{12}$ or $\frac{30}{40}$, &c. and the fraction $\frac{6}{12}$ is equal to the fraction $\frac{1}{2}$. For comparing, for instance, the fractions $\frac{3}{4}$ and $\frac{30}{40}$, in the latter the whole thing is conceived to be divided into ten times as many equal parts as in the former; each of which, therefore, is 10 times less than each of the former; and consequently if we take ten times as many of them as of the former, we shall take just the same quantity of the whole. And thus, the twelfth part of a foot being an inch, $\frac{6}{12}$ of a foot are 6 inches; but that is equal to half a foot, or to the fraction $\frac{1}{2}$. The principles laid down in this section are so simple, that by a few familiar illustrations a very young child may be able to comprehend them, yet upon these simple principles the whole doctrine of fractions depends.

89. Hence we see how we may easily bring a given fraction to lower terms, if its numerator and denominator be capable of being divided evenly by the same number. As any number which evenly divides another is said to *measure* it; so a number which evenly divides

two or more numbers is called a *common measure* of them. Numbers which admit no greater common measure than unity are said to be *prime* to each other; and if the terms of a fraction are prime to each other, it is in its lowest terms, as we cannot bring it to any equivalent fraction of lower terms.

Thus the fraction $\frac{5}{6}$ is in its lowest terms; and the fraction $\frac{4}{6}$ may be brought to its lowest terms by dividing both numerator and denominator by 2; for the equal fraction $\frac{2}{3}$ consists of numbers prime to each other.

90. Hence also it is easy to bring a given fraction (supposed to be in its lowest terms) to an equivalent one of another denominator, provided that other be some multiple of the given denominator.

Thus if it be required to bring $\frac{5}{6}$ to an equivalent fraction whose denominator shall be 18, we observe that in changing the denominator from 6 to 18, we multiply it by 3; and therefore to maintain the equality of the two fractions, we must multiply the numerator by 3, so that the required fraction is $\frac{15}{18}$. And if it be required to bring the same fraction $\frac{5}{6}$ to another whose denominator shall be 162, we only want to ascertain by what number 6 must be multiplied in order to give the product 162, that we may multiply the numerator 5 by the same number. This is ascertained by dividing 162 by 6, and we thus find that 5×27 is the required numerator. Thus also $\frac{4}{6}$ may be brought to a fraction whose denominator is 15; because 15 (though not a multiple of 6) is a multiple of 3, the denominator of the equal fraction $\frac{2}{3}$.

91. To bring a given fraction to its lowest terms, it is only necessary to divide both its terms by their greatest common measure, that is, by the greatest number which evenly divides them both.

Thus, if there be given the fractions $\frac{21}{42}$, it is plain that both its terms are evenly divisible by 3, or by 7, or by 21. But of these common measures 21 is the greatest, and will therefore give the smallest quotients: so that the lowest terms of the fraction are $\frac{1}{2}$. But if the terms of the given fraction be high numbers, we may be unable to ascertain by

inspection whether they be prime to each other; or if not, what number is their greatest common measure. We proceed therefore to state and demonstrate the method of discovering this.

92. Divide the greater number by the less; if there be no remainder, your divisor is the greatest common measure, inasmuch as no number greater than itself can measure the less of the two given numbers.

Thus, if the two given numbers be 12 and 96, 12 must be their greatest common measure; for it measures 96, and no number greater than 12 can measure 12.

93. But if there be a remainder on the first division, then divide your last divisor by that remainder; and so on, till you come to a remainder which will measure the last divisor. This remainder is the greatest common measure of the two given numbers: and therefore if you find no such remainder till you come to 1, the given numbers are prime to each other.

Thus, if the two given numbers be 182 and 559; dividing the greater by the less, we find the quotient 3 and the remainder 13: then dividing 182 by 13, we find the quotient 14, and no remainder: 13 the remainder, which measures the first divisor, is a common measure of 182 and 559, and their greatest common measure. First it is a common measure of them; for it measures 182, and therefore 3 times 182; and therefore $182\times3+13$, or the sum of 3 times 182 and 13. But that is equal to 559, as we have seen by the first division. Therefore it is a common measure of 182 and 559. But, secondly, it is their greatest common measure. For suppose any greater number, for instance 17, to be a common measure of 182 and 559. Since it measures 182, it must also measure 3 times 182: and since it measures 559, it measures $182\times3+13$, which is equal to 559. Inasmuch then as it measures both 182×3 and $182\times3+13$, it must measure 13; that is, a number greater than 13 must measure 13, which is absurd. Therefore 13 is the greatest common measure of 182 and 559.

94. A number consisting of a whole number, with a

fraction annexed, is termed a *mixed* number: as $5\frac{3}{7}$, $2\frac{4}{5}$, &c.

95. *A simple fraction* expresses one or more of the equal parts into which a unit is divided, without reference to any other fraction: as $\frac{1}{2}$, $\frac{7}{8}$, &c.

96. *A compound* fraction expresses one or more of the equal parts into which another fraction, or a mixed number, is divided.

Thus, $\frac{7}{8}$ of $\frac{10}{11}$, and $\frac{1}{3}$ of $\frac{4}{5}$ of $1\frac{4}{7}$, are compound fractions. Compound fractions have the word *of* interposed between the simple fractions of which they are composed.

97. *A complex fraction* is that which has a fraction either in its numerator or denominator, or in each of them.

Thus, $\frac{5\frac{1}{2}}{9}$, $\frac{8}{9\frac{2}{3}}$, &c. are complex fractions. The same operations can be performed on fractional as on integral quantities. Hence the doctrine of fractions comprises the *Addition*, *Subtraction*, *Multiplication*, and *Division* of Fractions. Before entering on these operations, it is proper to show how such quantities may be modified, without changing their value, so as to fit them for the several operations to be performed on them, and for the different uses to which they are to be applied. This will constitute *Reduction of Fractions.**

Reduction of Fractions.†

PROBLEM I.—*To find the greatest common measure of two given numbers.*

98. RULE.—Divide the greater number by the less.

* One great class of fractional quantities are called *decimal fractions* from the nature of their denominators. Hence, the doctrine of fractions is generally divided into two branches; one treating of fractions in general, and the other of the peculiar management, and properties of decimal fractions. Fractions which are not decimals are usually, although very improperly, called *vulgar*, or *common fractions.*

† Reduction of fractions consists of two branches; one which treats of the changes that may be made in the forms of abstract fractional quantities without changing their values; and the other which treats of fractions as referred to particular units, such as the denominations of money, weights, and measures. To the former branch belong problems II. IV. V. and VI.; and to the other, problems VII. &c.

If there be a remainder, divide the less by it; and thus proceed, always dividing the last divisor by the last remainder, till nothing remains. The divisor which leaves no remainder, is the common measure required.

If the divisor which leaves no remainder be unity or 1, then the given numbers are prime to each other; and therefore, can have no common measure greater than unity.

Ex. 1. Required the greatest common measure of 247 and 323.

In this example, the first remainder is 76; and the less number, 247, being divided by this, the remainder is 19. Dividing 76, the last divisor, by this, we find there is no remainder. Hence 19 is the common measure required, and would be found by division to be contained 13 times in 247, and 17 times in 323, both without a remainder. The reason of the rule is evident from § 92 and 93.

```
247)323(1
    247
    ---
     76)247(3
        228
        ---
         19)76(4
            76
```

Exercises.—Find the greatest common measures of the following numbers:

Ex. 1.	285 and 465.	Ans.	15.
2.	532 and 1274.	Ans.	14.
3.	888 and 2775.	Ans.	111.
4.	2145 and 3471.	Ans.	39.
5.	1879 and 2425.	Ans.	1.
6.	693 and 1815.	Ans.	33.
7.	2700 and 330.	Ans.	30.
8.	1827 and 1824.	Ans.	3.
9.	1776 and 1814.	Ans.	2.
10.	1800 and 1900.	Ans.	100.
11.	759 and 337.	Ans.	1.
12.	1014 and 1828.	Ans.	2.

PROBLEM II. *To reduce a fraction to its lowest terms.**

99. RULE I.—Divide the terms of the given fraction by any number that will measure both; the quo-

* A fraction is said to be in its *lowest terms*, or in its *simplest form*, when it is expressed in the least whole numbers possible.

tients will be the numerator and denominator of an equivalent fraction in lower terms. Let this fraction, if possible, be reduced in like manner; and proceed thus, till a fraction is obtained for whose terms no common measure can be found.

Ex. 2. Reduce $\frac{108}{162}$ to its lowest terms.

$$2)\frac{108}{162} \overset{9)}{=} \frac{54}{81} \overset{3)}{=} \frac{6}{9} = \frac{2}{3}. \quad \text{Ans.}$$

Here we divide the given terms by 2; those of the result by 9; and those of that result by 3; and we see that $\frac{108}{162}$ is equal to $\frac{54}{81}$, $\frac{6}{9}$, or $\frac{2}{3}$. The last of these has evidently no common measure greater than unity; it is therefore the expression required. The same result would have been obtained rather more quickly by dividing by 6 and 9.

The application of this rule will often be facilitated by the following directions and remarks: 1. If the terms of the fraction end in ciphers, cut an equal number from both. 2. If they end each in 5, or one in 5, and the other in a cipher, divide them both by 5; or double them and reject a cipher from each of the results. 3. If 2 measure the last figure of each term, it will measure the terms themselves. In like manner, if 4 measure the number expressed by the last two figures, or 8 that expressed by the last three, 4 in the one case, and 8 in the other, will measure both terms. 5. If 3 or 9 measure the sum or digits of each term, 3 in the one case, and 9 in the other, will measure both terms.

Ex. 3. Reduce $\frac{21000}{37800}$ to its simplest form.

$$6)\frac{210|00}{378|00} \overset{7)}{=} \frac{35}{63} = \frac{5}{9}. \quad \text{Ans.}$$

Here two ciphers are cut from the end of each term, which is equivalent to the division of each by 100. The quotients are then divided by 6, and the results by 7, and the fraction is reduced to the form $\frac{5}{9}$, which is its simplest form.

Exercises.—Reduce the following fractions to their simplest forms:

Ex. 1. $\frac{63}{105}$	Ans. $\frac{3}{5}$		Ex. 6. $\frac{57}{63}$	Ans. $\frac{19}{21}$	
2. $\frac{55}{121}$	$\frac{5}{11}$		7. $\frac{176}{1000}$	$\frac{22}{125}$	
3. $\frac{120}{320}$	$\frac{3}{8}$		8. $\frac{960000}{1152000}$	$\frac{5}{6}$	
4. $\frac{240}{312}$	$\frac{10}{13}$		9. $\frac{144000}{1920000}$	$\frac{3}{40}$*	
5. $\frac{126}{162}$	$\frac{7}{9}$				

100. Rule II.—Find by problem I. the greatest common measure of the numerator and denominator. Divide them both by this, and the quotients will be the numerator and denominator of the required fraction.

It is often of advantage to carry the reduction as far, by the first rule, as can readily be done, and then to apply the second rule to the result.

Ex. 4. Reduce the fraction $\frac{679}{1746}$ to its lowest terms.

In this example, by dividing the denominator by the numerator, the numerator by the remainder, &c. we find the greatest common measure to be 97: and dividing both terms of the given fraction by this, we obtain $\frac{7}{18}$, which is the equivalent fraction in its lowest terms.†

Ex. 5. Reduce $\frac{8280}{11385}$ to its lowest terms.

Here, because the terms end in 5 and 0, 5 is a measure, and reduces the fraction to $\frac{1656}{2277}$. This again is reducible by 9, because 9 measures the sum of the digits of each term; and the result is $\frac{184}{253}$, which, by the second rule, is reducible to $\frac{8}{11}$.

* This method of reducing fractions to their lowest terms is very convenient and easy in practice, when the terms of the proposed fractions are not very large, or when the divisors are readily discovered. It labours under the disadvantage, however, of not determining, in many cases, whether the fraction is in its lowest terms or not; and besides its being a process which depends on trial, and which, therefore, is not strictly mathematical, the measures are generally discovered with difficulty, unless they are some of the smaller numbers. Thus, if the fraction $\frac{399}{627}$ were proposed, we should readily discover that it may be reduced, by division by 3, to the form $\frac{133}{209}$, which we would perhaps conceive to be its simplest form, not knowing that it is still farther reducible by 19, and that the simplest form is $\frac{7}{11}$. The method in the next Rule, though often tedious, is perfect in principle, always reducing the fraction to its simplest form, and not depending on trial.

† It may be proper for the student to observe, that, at the concl... of every operation, if there be a fraction, it should be red... simplest form; see page 45 and 46.

Exercises.—Reduce the following fractions to their lowest terms:

Ex. 1. $\frac{468}{1266}$	Ans. $\frac{78}{211}$	Ex. 6. $\frac{1236}{1632}$	Ans. $\frac{103}{136}$
2. $\frac{272}{425}$	$\frac{16}{25}$	7. $\frac{285714}{999999}$	$\frac{2}{7}$
3. $\frac{594}{2142}$	$\frac{33}{119}$	8. $\frac{10395}{16808}$	$\frac{945}{1528}$
4. $\frac{5170}{6754}$	$\frac{235}{307}$	9. $\frac{5415}{38505}$	$\frac{361}{2567}$
5. $\frac{6465}{7335}$	$\frac{431}{489}$	10. $\frac{15863}{21489}$	$\frac{547}{741}$

PROBLEM III.*—*To find the least common multiple of two or more given numbers.*

101. RULE I.—Arrange the given numbers in succession, and find by inspection a number which will measure as many of them as possible. By this number divide all the given numbers which it measures, and write the quotients and the new divided numbers in a new line. Proceed, if possible, in the same manner with the numbers in this line; and thus continue the process till no number greater than a unit will measure any two or more of the numbers last found. Then multiply all the numbers in the last line, and all the divisors employed in the operations, continually together, and the result will be the common multiple required.

The work is much shortened by rejecting in any line any number that is contained without remainder in any other number in the line. If no two of the given numbers have any common measure greater than unity, their continual product will be the least common multiple.

Ex. 6. Required the least common multiple of 24, 10, 9, 32, 6, 45, and 25.

2)24	10	(9)	32	(6)	45	25
3)12	(5)		16		45	25
5)(4)			16		15	25
			16		3	5

Ans. $2 \times 3 \times 5 \times 16 \times 3 \times 5 = 7200$

* This problem and problem I. are given in this place because, though

Here, the given numbers being placed in the same line; as we see by inspection that 9 is contained exactly in 45, and 6 in 24, these are neglected. Then using 2 as a divisor, we obtain the quotients, 12, 5, and 16, and we place in the line with them the undivided numbers, 45, and 25: and since the quotient 5 is contained exactly in either of these, it is rejected. We then divide by 3, and obtain the quotients 4 and 15, which with the undivided numbers, 16 and 25, are placed in a new line;—4 is then rejected, as it is a measure of 16; and dividing by 5, we place in the next line the quotients 3 and 5, and the undivided number 16. Then as no two of these have any common measure greater than unity, the three divisors and the three numbers in the last line are multiplied continually together, and the product, 7200, is the common multiple required.*

102. Rule II. Find by Problem I. the greatest common measure of two of the given numbers. By this divide one of those two numbers, and multiply the quotient by the other. Perform a similar operation on the product and another of the given numbers. Continue the process thus with each successive quotient, till all the numbers have been used, and the final result will be the least common multiple required.†

they respect whole numbers, they are chiefly useful in the second and fourth problems in this article.

* With respect to *the reason of this rule*, it is difficult to give a strict, and at the same time an easily comprehended proof; and for most learners the following illustration may perhaps be preferable:

In the operation $2\times12\times5\times16\times45\times25$, the product of the first divisor and the numbers in the second line is evidently a multiple of each of the given numbers, 24 being contained in it $5\times16\times45\times25$ times; 10, the second number, $12\times16\times45\times25$ times, &c. Again, $2\times3\times4\times16\times15\times25$, the product of the first two divisors and the numbers in the third line, is also a multiple of each of the given numbers, 24 being contained in it $16\times15\times25$ times; 10, the second number, $3\times4\times16\times3\times25$ times, (since $2\times15=30$, and $30=10\times3$,) 32, the fourth number, $3\times4\times15\times25$ times, &c.: and the illustration may be extended in a similar manner to the rest of the operation, which will readily appear from considering that 6 is contained exactly 4 times in 24, and will therefore be contained without remainder in any multiple of 24, and 4 times as often as 24. In like manner it would appear that 9 may be rejected, as 45 is a multiple of it. That 5 and 4 may be rejected in the succeeding lines, will be evident from this, that they would disappear were the lines that contained them divided respectively by 5 and 4.

† It may be remarked with respect to the practical application of these rules, that the second always gives the *least* common multiple. The

Let us take the last example: rejecting 9 and 6 as before, we divide 24 by 2, the greatest common measure of it and 10, and we multiply the quotient 12 by 10, the product, 120 is the least common multiple of 24 and 10. Then, the greatest common measure of 120 and 32 being 8, we have $32 \div 8 = 4$, and $120 \times 4 = 480$, the least common multiple of 24, 10, and 32. In the next place, the greatest common measure of 480 and 45 is 15; and, since $45 \div 15 = 3$, we have $480 \times 3 = 1440$, the least common multiple of 24, 10, 32, and 45. Lastly, since the greatest common measure of 1440 and 25 is 5, and since $25 \div 5 = 5$, we have $1440 \times 5 = 7200$, the least common multiple of 24, 10, 32, 45, and 25, and consequently of 9 and 6, of which 45 and 24 are multiples.*

multiple found by the first is not always the *least* possible; but it will be such if care is always taken to use such a divisor as will measure more of the given numbers than any other divisor would. This rule, therefore, being very easy in practice, is generally preferred to the other. It may be farther remarked, that by practice the pupil will gradually become able to discover, in many instances, the common multiples by inspection, especially when the given numbers are not large.

The least common multiple of any number of quantities, literal or numeral, may be easily found thus:

Resolve each quantity into its simplest factors, putting the product of equal factors, when there are any, in the form of powers; then multiply together the highest powers of all concerned, and the product will be the least common multiple required.—See my "Treatise on Algebra, theoretical and practical," p. 72, 2d edition.

* The proof of this rule depends on the principle that *if the product of any two numbers be divided by any factor which is common to both, the quotient will be a common multiple of the two numbers.* Thus if 48, the product of 6 and 8, be divided by 2, a factor of both, the quotient 24 will be a multiple of both, since it may be regarded either as 8 multiplied by the quotient of 6 by the factor 2, or as 6 multiplied by the quotient of 8 by the same factor. Now this being so, it is obvious that the greater the common measure is, the less will be the multiple; and consequently the greatest common measure will produce the least common multiple. When the common multiple of the first two numbers is found, it is evident that any number which is a common multiple of it and of the third number, will be a multiple of the first, second, and third numbers; and thus the reason of every part of this rule is manifest.

Exercises.

Required the least common multiples of the following numbers:

Ex. 1.	6	10	15	18					Ans.	90.
2.	7	11	13	3						3003.
3.	8	12	20	24	25					600.
4.	63	12	84	7						252.
5.	54	81	63	14						1134.
6.	2	3	4	5	6	7	8	9		2520.

Problem IV. *Any number of fractions having different denominators being given, to find equivalent fractions having a common denominator.*

103. Rule I. Find by Problem III. a common multiple of all the denominators: this will be the common denominator. Then divide the common multiple by the first of the given denominators, and multiply the quotient by the first of the given numerators: the product will be the first of the required numerators. The other numerators are found in a similar manner.

104. Rule II. Multiply each numerator by all the denominators except its own, and the product will be the numerator of the equivalent fraction. Multiply all the denominators continually together for the common denominator.*

* With respect to *the reason of these rules*, it is evident that the operation by the second is nothing else than the multiplication of both the terms of each fraction by all the denominators except its own; and in this way the rule might be expressed. Thus, the first of the given fractions is $\frac{5}{8}$, (see Ex. 7.) which by the operation is transformed into $\frac{5\times12\times18\times20}{8\times12\times18\times20}$, or $\frac{21600}{34560}$; and this by the property proved in § 57, is equal to the proposed fraction $\frac{5}{8}$.

In the first rule, the division of the common multiple by the given denominator is evidently nothing else than the finding of the number by which if both the numerator and the denominator of the given fraction be multiplied, the denominator of the result will be the same as the common multiple; and by the principle before referred to, the result will be equal to the given fraction. Thus, in the example above alluded to, when 360 is divided by 8, the quotient is 45, by which if both terms of the fraction $\frac{5}{8}$ be multiplied, there results $\frac{225}{360}$ for the equivalent fraction required.

Ex. 7. Reduce $\frac{5}{8}$, $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{9}{20}$ to fractions having a common denominator.

8)360	12)360	18)360	20)360
45	30	20	18
5	7	11	9
225	210	220	162

Ans. $\frac{225}{360}$, $\frac{210}{360}$, $\frac{220}{360}$, $\frac{162}{360}$.

By Rule I.—Here, by Problem III., the least common multiple is found to be 360; and the rest of the work will stand as in the example. The correctness of the result would be proved by reducing them to their simplest forms, as the given fractions would thus be obtained. Thus $\frac{225}{360}$ would be reduced to $\frac{5}{8}$, $\frac{210}{360}$ to $\frac{7}{12}$, &c.

By Rule II.—$5\times12\times18\times20=21600$, the first numerator; $7\times8\times18\times20=20160$, the second numerator; $11\times8\times12\times20=21120$, the third numerator; $9\times8\times12\times18=15552$, the fourth numerator; and $8\times12\times18\times20=34560$, the common denominator. Hence the equivalent fractions found by this rule are $\frac{21600}{34560}$, $\frac{20160}{34560}$, $\frac{21120}{34560}$, $\frac{15552}{34560}$, which being reduced to their lowest terms, would in like manner, be shown to be equivalent to the given fractions.*

Exercises.—Reduce the following sets of fractions to other equivalent fractions having the least common denominator :

Exercises.	*Answers.*
1. $\frac{5}{12}$, $\frac{9}{16}$, $\frac{11}{24}$, $\frac{17}{33}$, $\frac{3}{8}$.	$\frac{220}{528}$, $\frac{297}{528}$, $\frac{242}{528}$, $\frac{272}{528}$, $\frac{198}{528}$.
2. $\frac{11}{27}$, $\frac{19}{24}$, $\frac{5}{6}$, $\frac{7}{15}$, $\frac{3}{5}$, $\frac{1}{9}$.	$\frac{440}{1080}$, $\frac{855}{1080}$, $\frac{900}{1080}$, $\frac{504}{1080}$, $\frac{648}{1080}$, $\frac{120}{1080}$.
3. $\frac{17}{80}$, $\frac{19}{60}$, $\frac{11}{60}$, $\frac{53}{75}$, $\frac{4}{9}$, $\frac{5}{36}$.	$\frac{765}{3600}$, $\frac{1140}{3600}$, $\frac{660}{3600}$, $\frac{2544}{3600}$, $\frac{1600}{3600}$, $\frac{500}{3600}$.
4. $\frac{11}{35}$, $\frac{19}{45}$, $\frac{25}{42}$, $\frac{1}{100}$, $\frac{79}{81}$.	$\frac{17820}{56700}$, $\frac{23940}{56700}$, $\frac{23750}{56700}$, $\frac{567}{56700}$, $\frac{55300}{56700}$.

* In this example the results found by the second rule are expressed inconveniently large, and the same is very generally the case. Hence the other rule should always be preferred, except when the given denominators are *prime to each other*; that is, when they have no common measure greater than a unit.

Exercises.	*Answers.*
5. $\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}$.	$\frac{693}{3465}, \frac{495}{3465}, \frac{385}{3465}, \frac{315}{3465}$.
6. $\frac{11}{12}, \frac{17}{18}, \frac{29}{30}, \frac{47}{48}, \frac{7}{16}$.	$\frac{660}{720}, \frac{680}{720}, \frac{696}{720}, \frac{705}{720}, \frac{315}{720}$.
7. $\frac{17}{100}, \frac{17}{1000}, \frac{17}{10000}$.	$\frac{1700}{100000}, \frac{170}{100000}, \frac{17}{100000}$.
8. $\frac{41}{60}, \frac{13}{90}, \frac{13}{25}, \frac{1}{105}, \frac{4}{9}$.	$\frac{4305}{6300}, \frac{910}{6300}, \frac{3276}{6300}, \frac{60}{6300}, \frac{2800}{6300}$.
9. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \frac{16}{243}, \frac{32}{729}$.	$\frac{243}{729}, \frac{162}{729}, \frac{108}{729}, \frac{72}{729}, \frac{48}{729}, \frac{32}{729}$.

Problem V.—*To reduce an improper fraction to a whole or mixed number.*

105. Rule.—Divide the numerator by the denominator; the quotient will be the whole number required; and if there be any remainder, write it over the given denominator for the fractional part of the required result.*

Ex. 1. Reduce $\frac{56}{7}$ and $\frac{200}{9}$ to whole or mixed numbers.

$$\frac{56}{7} \qquad 7)56 \qquad 8, \text{ Ans.} \qquad\qquad \frac{200}{9} \qquad 9)200 \qquad 22\tfrac{2}{9}, \text{ Ans.}$$

Exercises.—Reduce the following fractions to whole or mixed numbers:

Ex. 1.	$\frac{19}{19}$	Ans. 1.	Ex. 4.	$\frac{10000}{111}$	Ans. $90\frac{10}{111}$.
2.	$\frac{6437}{298}$	$21\frac{179}{298}$.	5.	$\frac{509}{12}$	$42\frac{5}{12}$.
3.	$\frac{7536}{375}$	$20\frac{12}{125}$.	6.	$\frac{750}{25}$	30.

Problem VI.—*To reduce a whole or mixed number to an improper fraction.*

106. Rule.—Multiply the whole number by the denominator of the fractional part, and to the product add the numerator; the sum will be the required numerator; below which write the given denominator.

A whole number may be expressed in a fractional form by writing a unit below it; or by multiplying it by any whole number, and writing that number below the product as denominator. Thus $9 = \frac{9}{1} = \frac{36}{4}$, &c.

* The reason of this rule and of that for the next problem, is evident from the nature of fractions.

Ex. 9. Reduce $5\frac{11}{12}$ to an improper fraction.

$$\begin{array}{r} 5\frac{11}{12} \\ 12 \\ \hline 71 \\ \hline 12 \end{array} \quad \text{Ans.}$$

Exercises.—Reduce the following numbers to improper fractions;

Ex. 1. $15\frac{1}{6}$	Ans. $\frac{91}{6}$	Ex. 4. $21\frac{117}{583}$	Ans. $\frac{12360}{583}$
2. $3\frac{17}{55}$	$\frac{182}{55}$	5. $1\frac{999}{1000}$	$\frac{1999}{1000}$
3. $46\frac{5}{8}$	$\frac{373}{8}$	6. $19\frac{5}{7}$	$\frac{138}{7}$

Problem VII.—*To express any given quantity as a fraction of another given quantity of the same kind, considered as an integer.*

107. Rule.—Make the integer the denominator of the required fraction, and the other given quantity its numerator, both being reduced to the same denomination, if they be not of the same already.

Ex. 10. Reduce 13*s*. 9*d*. to the fraction of £1.

In 13*s*. 9*d*. there are 165 pence, and in £1, 240 pence; the fraction therefore is £$\frac{165}{240}$, which, by reduction to its lowest terms, becomes £$\frac{11}{16}$. The reason of this is evident, since 1 penny is $\frac{1}{240}$ of a pound, and in 13*s*. 9*d*. there are 165 pence, or $\frac{165}{240}$ of a pound.

Ex. 11. Express a pound troy weight as a fraction of a pound avoirdupois.

A pound avoirdupois weight contains 7000 grains, and a pound troy weight 5760 grains. (*see page* 54.) Hence the fraction is $\frac{5760}{7000}$, or in its lowest terms $\frac{144}{175}$.*

Exercise 1. Reduce 12*s*. 6*d*. to the fraction of £1.

Ans. $\frac{5}{8}$.

Ex. 2. Reduce £32 10*s*. to the fraction of £100.

Ans. $\frac{13}{40}$.

Ex. 3. Reduce 1*qr*. 20*lb*. to the fraction of 1*cwt*.

Ans. $\frac{3}{7}$.

* This rule enables us to find the ratio of two quantities. Thus, the ratio of 13*s*. 9*d*. to 1*l*. is that of $\frac{11}{16}$ to 1, or 11 to 16; and a pound avoirdupois is to a pound troy, as 1 to $\frac{144}{175}$, or as 175 to 144.

Ex. 4. Reduce 2 hours to the fraction of 23*h*. 56*m*. 4*s*.

Ans. $\frac{1800}{21541}$.

Ex. 5. Reduce 2*s*. and 1*d*. to the fraction of 6*s*. 8*d*.

Ans. $\frac{5}{16}$.

Ex. 6. The height of Ben Nevis, the highest mountain in Britain, is 4350 feet, and that of Mount Ararat 9500 feet: express the former as a fraction of the latter in its simplest form. Ans. $\frac{87}{190}$.

Ex. 7. Express 96 pages as a fraction of a book containing 432 pages. Ans. $\frac{2}{9}$.

Ex. 8. If one farm contains 27 acres, 2 roods, 14 poles, and another 160 acres, 28 poles; what is the simplest form of the fraction expressing their comparative magnitudes?

Ans. $\frac{97}{482}$.

Ex. 9. The height of the highest peak of the Himmaleh mountains is supposed to be 27600 feet, and that of Chimborazo, the highest peak of the Andes, in South America, 21440 feet above the level of the sea: express the latter as a fraction of the former in its simplest form. Ans. $\frac{268}{345}$.

Problem VIII.—*To find the value of a fraction in the denominations contained in the integer.*

108. Rule.—Consider the numerator as expressing the integer, taken as often as there are units in the numerator, and divide it by the denominator.*

Example 12. Required the value of £$\frac{4}{7}$.

Here the integer is £1; and the numerator being regarded as 4 times that integer, or £4, is divided by the denominator: the quotient 11*s*. 5$\frac{1}{7}$*d*. is the value of the fraction. The reason of the rule is manifest from the nature of fractions.

£	*s*.	*d*.	
7)4	0	0	
0	11	5$\frac{1}{7}$,	Ans.

Ex. 13. The height of the Antesana hamlet, near Quito, South America, *the highest inhabited spot on the surface of the globe*, is about 13400 feet: if a person have ascended through $\frac{3}{8}$ of this space, to what height has he ascended?

* This problem is, in strictness, a particular case of the next. On account of its frequent use, however, it is better in a separate form.

Here, the height of the hamlet is the integer; and the numerator expressing three times the quantity, we multiply by 3 and divide by 8, and we find for the answer 5025 feet.

$$\begin{array}{r} 13400 \text{ feet.} \\ 3 \\ \hline 8)\,40200 \\ \hline \text{Ans. } 5025 \text{ feet.} \\ \hline \end{array}$$

Exercise 1. Required the value of £$\frac{5}{9}$. Ans. 11*s.* $1\frac{1}{3}$*d.*

Ex. 2. Required the value of $\frac{6}{13}$ of a foot.
Ans. 5 *in.* $6\frac{6}{13}$ *lines.*

Ex. 3. Required the $\frac{3}{5}$ of 756 dollars, 25 cents.
Ans. \$453·75.

Ex. 4. Required the value of $\frac{5}{6}$ of 175 tons.
Ans. 145 *tons*, 16*cwt.* 2*qr.* $18\frac{2}{3}$*lb.*

Ex. 5. The Earth revoles on its axis in 23 hours, 56 minutes, 4 seconds; in what times does it perform $\frac{4}{9}$ of a rotation? Ans. 10 *hours*, 38 *min.* $15\frac{1}{9}$ *sec.*

Ex. 6. By the articles of the union of Great Britain and Ireland, which took place in 1801, during the first 20 years, Britain was to contribute $\frac{15}{17}$, and Ireland $\frac{2}{17}$, of the amount of the public expenditure. How much did each country contribute in making a million sterling?
Ans. £882352 18*s.* $9\frac{15}{17}$*d.* and £117647 1*s.* $2\frac{2}{17}$*d.*

PROBLEM IX.—*To reduce a given fraction to another of a lower denomination.*

109. RULE.—As in common reduction, reduce the given numerator, considered as an integer, to the denomination to which the fraction is to be reduced, and write below it the given denominator.

Example 14. Reduce $\frac{5}{1600}$ dollar to the fraction of a cent.

Here by reducing 5 dollars to cents, and writing 1600 below it, we have for the required fraction $\frac{500}{1600}$ cent, which, by reduction to its lowest terms, becomes $\frac{5}{16}$ cent.*

Exercise 1. Reduce £$\frac{1}{15}$ to the fraction of a shilling.
Ans. $\frac{4}{3}$ shilling.

* The reason of this is manifest, since by the nature of fractions, the given fraction expresses a sixteen hundredth part of 5 dollars, and must therefore express a sixteen hundredth part of the cents in 5 dollars.

Ex. 2. Reduce $\frac{3}{20}$ hundred weight to the denomination of pounds. Ans. $16\frac{4}{5}$*lb*.

Ex. 3. What part of a second is $\frac{1}{1000000}$ day ? Ans. $\frac{54}{625}$.

Ex. 4. What part of a dollar is $\frac{5}{9}$ cent ? Ans. $\$\frac{1}{180}$.

The converse of this problem, or the reduction of a fraction to a higher denomination is seldom of use. It is performed by operating on the *denominator* in the way mentioned in the rule. Thus, if it be required to reduce $\frac{9}{20}$ pence to the fraction of a pound, let the denominator be multipled by 20 and 12, and the numerator be retained: the result £$\frac{9}{4800}$, or, in its simplest form, £$\frac{3}{1600}$ is the result required.*

Addition of Fractions.

110. Rule.—Reduce the given fractions to others having a common denominator, if they be not such already. When they are in this state add all the numerators together, and below their sum write the common denominator: the result will be the sum of the fractions; which, if it be an improper fraction, must be reduced to a whole or mixed number.†

If some of the given quantities be mixed numbers, the fractions are to be added by the preceding part of the rule;

* The method of reducing compound to simple fractions will be given in Multiplication of Fractions, and that of reducing complex fractions to simple ones, in Division of Fractions. These are the natural and proper positions of these problems; but should any teacher conceive their introduction necessary here, with a view to prepare fractional quantities for Addition, Subtraction, &c. it will be easy for the learner to turn over for them to Multiplication and Division.

† When the fractions whose sum we want to find have the same denominator, the method of performing the operation is as obvious as the addition of whole numbers. For it is as plain that the sum of two ninths and five ninths is seven ninths, (that is, $\frac{2}{9}+\frac{5}{9}=\frac{7}{9}$,) as that the sum of 2 dollars and five dollars is equal to seven dollars. Ninths in the former case, and dollars in the latter, are but the denomination, of the numbers which we add: and in place of the fractional notation, the columns in which the numbers 2 and 5 stand might be headed with the denomination ninths, as it is commonly with the denomination dollars. And if the fractions which we are required to add have different denominators, they can be brought by reduction of fractions, to equivalent fractions of the same denominator; and the same reasoning will apply as before.

and the whole numbers, with any integral part that may be obtained by the adding of the fractions, are to be added by addition of whole numbers.

Example 1. Add together $\frac{14}{15}$, $\frac{11}{15}$, and $\frac{8}{15}$.

Here the sum is $\frac{33}{15}$, which becomes, by reduction, to a mixed number, $2\frac{3}{15}$, or, by the reduction of the fractional part to its lowest terms, $2\frac{1}{5}$, the sum required.

$$\left.\begin{array}{r}14\\11\\8\end{array}\right\}15$$

$$\begin{array}{l}\underline{33}\\ 15)33\,(2\tfrac{3}{15}\text{ or }2\tfrac{1}{5}\\ \underline{30}\ \text{Ans.}\\ 3\end{array}$$

Example 2. Add together $3\frac{31}{40}$, $\frac{29}{36}$, and $6\frac{23}{25}$.

$$\begin{array}{rcl}3\frac{31}{40} & \ldots & 1395\\ \frac{29}{36} & \ldots & 1450\\ 6\frac{23}{25} & \ldots & 1656\end{array}\Bigg\}\,1800$$

$$\text{Ans. } 11\tfrac{901}{1800} \qquad 4501$$

$$\begin{array}{l}1800)4501\,(2\tfrac{901}{1800}\\ \underline{3600}\\ 901\end{array}$$

In this example, by reducing the given fractions to equivalent ones having a common denominator, we find the first to be $\frac{1395}{1800}$, the second $\frac{1450}{1800}$, &c. Then by adding the numerators, we find the sum of the fractions to be $\frac{4501}{1800}$, or $2\frac{901}{1800}$, by reduction to a mixed number. We then write the fractional part beneath the given fractions; and carrying 2 to the whole numbers, we find the required sum to be $11\frac{901}{1800}$.

Exercises.

1. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\times\frac{5}{6}+\frac{6}{7}$. Ans. $2\frac{79}{84}$.
2. $90\frac{3}{4}+67\frac{2}{3}+4\frac{17}{20}+15\frac{5}{12}+34\frac{14}{87}$. Ans 212. $\frac{1469}{1740}$.
3. $5\frac{9}{10}+73\frac{3}{8}+37\frac{1}{10}+\frac{19}{70}+1\frac{31}{48}+2\frac{81}{80}$.

Ans. $120\frac{488}{504}$.

4. $\frac{8}{15}+\frac{9}{20}$. Ans. $\frac{59}{60}$.
5. $\frac{7}{8}+\frac{1}{12}+\frac{17}{18}+\frac{23}{24}+\frac{20}{27}$. Ans. $3\frac{65}{108}$.
6. $\frac{7}{8}+\frac{7}{12}+\frac{13}{16}+\frac{11}{18}+\frac{19}{24}$. Ans. $3\frac{195}{304}$.
7. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}$. Ans. $\frac{127}{128}$.
8. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\frac{31}{32}+\frac{63}{64}+\frac{127}{128}$. Ans. $6\frac{1}{128}$.
9. $\frac{1}{3}+\frac{4}{7}+\frac{31}{32}$. Ans. $1\frac{587}{672}$.
10. $\frac{5}{8}+\frac{5}{14}+\frac{1}{6}$. Ans. $1\frac{25}{168}$.
11. $3\frac{5}{8}+12\frac{1}{6}+13\frac{83}{84}$. Ans. $29\frac{131}{168}$.
12. $35\frac{9}{10}+21\frac{11}{19}+44\frac{3}{5}+8\frac{99}{101}$. Ans. $111\frac{2273}{3838}$.
13. A man left a legacy of 12000 dollars among two sons and a daughter, so that one son should have $\frac{1}{2}$ of it, and the other $\frac{1}{3}$ of it. What proportion of the legacy did the sons receive, and how much?

Ans. $\frac{5}{6}$ and they received $10000.

Subtraction of Fractions.

111. Rule.—Set the less quantity below the greater, and prepare them as in Addition of Fractions. Then, if possible, subtract the lower numerator from the upper; below the remainder write the common denominator, and if there be whole numbers, find their difference as in Simple Subtraction. But if the lower numerator exceed the upper, subtract it from the common denominator; to the remainder add the upper numerator; write the common denominator beneath the sum; and carry one to the whole number in the lower line.*

Exam. 1.		2.			3.		
From	$87\frac{8}{11}$	$73\frac{5}{6}$	. . 25	$\left.\begin{matrix}\\ \\ \end{matrix}\right\}$ 30	$10\frac{2}{25}$	. . 14	$\left.\begin{matrix}\\ \\ \end{matrix}\right\}$ 175
Take	$42\frac{6}{11}$	$17\frac{4}{15}$	. . 8		$3\frac{22}{35}$	. . 110	
Rem.	$45\frac{5}{11}$	$56\frac{17}{30}$	17		$6\frac{79}{175}$	79	

Exercises.

1. What is the difference of $\frac{9}{14}$ and $\frac{17}{21}$? Ans. $\frac{1}{6}$.

* The reason of this rule is evident, from the explanation already given of Simple Subtraction, and of Addition of Fractions.

2. What is the difference between $10\frac{7}{8}$ and $12\frac{1}{2}$? Ans. $1\frac{5}{8}$.

3. What is the excess of $\frac{7}{8}$ above $\frac{3}{8}$? Ans. $\frac{1}{2}$.

4. What is the excess of $\frac{23}{42}$ above $\frac{11}{126}$? Ans. $\frac{7}{9}$.

5. What is the excess of $12\frac{3}{4}$ above $10\frac{13}{16}$? Ans. $1\frac{15}{16}$.

6. What is the excess of $\frac{1}{7}$ above $\frac{1}{11}$? Ans. $\frac{4}{77}$.

7. $12\frac{23}{28}-9\frac{27}{35}=3\frac{1}{20}$.

8. $4\frac{1}{24}-3\frac{1}{16}=\frac{47}{48}$.

9. What is the difference between the sum of $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ and the sum of $\frac{4}{5}+\frac{2}{3}+\frac{11}{15}$? Ans. $1\frac{5}{12}$.

10. A man left a legacy of £10000 sterling among three sons, so that the eldest should have $\frac{1}{2}$ of it, and the second $\frac{1}{3}$ of it. What proportion of the legacy did the youngest receive? Ans. $\frac{1}{6}$, or £1666 13*s*. 4*d*.

Multiplication of Fractions.

112. Rule.—If any of the quantities be whole or mixed numbers, reduce them to improper fractions. Find the product of the numerators for the numerator of the required result, and the products of the denominators for its denominator.

As a contraction, if a numerator and any denominator be the same, reject them; or divide a numerator and any denominator by any number that will measure them, and instead of them employ the results.

Compound fractions are reduced to simple ones in the same manner.

When a fraction is to be multiplied by a whole number, the numerator is multiplied by the integer and the denominator retained; or, which amounts to the same thing, whenever we have to multiply a fraction by an integer which measures its denominator, the product is exhibited in lower terms by dividing the denominator by the integer, than by multiplying its numerator. Thus, in the first place, it is as plain that 3 times $\frac{9}{10}$ is $\frac{27}{10}$, or $2\frac{7}{10}$, as that 3 times 9 is 27. And, again, 3 times $\frac{2}{9}$ is by the one process $\frac{6}{9}$; by the other $\frac{2}{3}$: results which we know are equal from § 88.

Example 1. Multiply $\frac{7}{8}$ by $\frac{12}{19}$.

Ans. $\frac{7}{8}\times\frac{12}{19}=\frac{84}{152}=\frac{21}{38}$.

Here by taking the products of the numerators, and of the denominators, we find for the required result $\frac{84}{152}$, or, in its simplest form. $\frac{21}{38}$.*

2. Multiply $\frac{4}{7}$ by $\frac{7}{11}$. Ans. $\frac{4}{7}\times\frac{7}{11}=\frac{28}{77}=\frac{4}{11}$.

In this example as one of the numerators is 7, and also one of the denominators the same, by neglecting this numerator and denominator, we have $\frac{4}{11}$ for the required result, the same as before.

$$\frac{4}{(7)}\times\frac{(7)}{11}=\frac{4}{11}.$$

3. Multiply $17\frac{5}{6}$ by $12\frac{3}{4}$.

These quantities, by reduction to improper fractions, $\frac{107}{6}$ and $\frac{51}{4}$. Thus, $\frac{107}{6}\times\frac{51}{4}=\frac{5457}{24}=\frac{1819}{8}$, by reduction to its lowest terms; and this being reduced to a mixed number, becomes $227\frac{3}{8}$, the product required.†

* The *reason of this rule* will appear evident, from the principles established in the beginning of the present chapter. Thus, since $\frac{12}{19}$ is the same as one nineteenth of 12, it is plain that the product of $\frac{7}{8}$ by $\frac{12}{19}$ is $\frac{1}{19}$ of its product by 12. Hence it appears, that to find the required product, we are to multiply $\frac{7}{8}$ by 12 and to divide the result by 19; and by the principles referred to, this will be effected by multiplying the numerator by 12, and the denominator by 19; that is, the numerator of the final result is the product of the given numerators, and its denominator the product of the given denominators. The contraction evidently produces the same result that would be obtained if the terms of the product found by the rule, were divided by the numbers rejected, or by the common measure employed in the contraction.

From what we have seen above, it appears, that in every case in what is called Multiplication of Fractions, there is not only a *multiplication* but also a *division*. Hence, in the first example, since we multiply $\frac{7}{8}$ by 12, and divide the product by 19, the result must be obviously less than the multiplicand $\frac{7}{8}$. It therefore follows, that the product of a quantity by a fraction must be always less than the quantity itself. It likewise follows, that when one factor is less than an unit, the product is less than the other, and consequently, when both factors are less than units, the product is less than either.

† The product may also be found as in the margin, without reducing the factors to improper fractions. In this mode, 12 times $\frac{5}{6}=\frac{60}{6}=10$, which is carried to the product of 17 and 12. We have then to find the product of 17 and $\frac{3}{4}$, and $\frac{5}{6}$ and $\frac{3}{4}$. The former is $12\frac{3}{4}$, and the latter $\frac{5}{8}$. The sum of all the three partial products is $227\frac{3}{8}$, the same as the answer found already.

$$\begin{array}{r l}
17\frac{5}{6} & \\
12\frac{3}{4} & \\
\hline
214 & \\
12\frac{3}{4}\ .\ .\ 6 & \left.\right\} 8 \\
\frac{5}{8}\ .\ .\ 5 & \\
\hline
227\frac{3}{8} & \frac{11}{8},\ \text{or}\ 1\frac{3}{8}
\end{array}$$

4. Reduce the compound fraction $\frac{3}{4}$ of $\frac{8}{9}$ of $3\frac{1}{3}$ to a simple fraction.

Here the mixed number $3\frac{1}{3}$ is reduced to $\frac{10}{3}$. Then by taking the product of the numerators, 3, 8, and 10; and also that of the denominators, 4, 9, and 3, we obtain $\frac{240}{108}$, or in its lowest terms $\frac{20}{9}$, the simple fraction required, which might be reduced to the mixed number $2\frac{2}{9}$. In this example the work might be contracted by rejecting the numerator and denominator 3, and by dividing the denominator 4 and the numerator 8 by 4, and the same result would be obtained.

Exercises.

1. $\frac{5}{12}\times\frac{13}{17}=\frac{65}{204}$.
2. $\frac{12}{13}\times\frac{7}{8}=\frac{21}{26}$.
3. $\frac{11}{15}\times\frac{29}{33}=\frac{29}{45}$.
4. $\frac{9}{14}\times\frac{3}{17}=\frac{27}{238}$.
5. $\frac{15}{79}\times\frac{19}{20}=\frac{57}{316}$.
6. $19\frac{7}{19}\times1\frac{5}{77}=20\frac{916}{1463}$.
7. $21\frac{5}{6}\times21\frac{5}{7}\times21\frac{5}{8}=10252\frac{13}{42}$.
8. $\frac{3}{17}\times1\frac{9}{11}\times\frac{4}{9}=\frac{80}{561}$.
9. $\frac{5}{7}\times9\frac{2}{3}=6\frac{19}{21}$.
10. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}=\frac{3}{7}$.
11. $\frac{7}{8}$ of $\frac{5}{6}$ of $7=5\frac{5}{48}$.
12. $\frac{5}{9}\times\frac{7}{10}+\frac{2}{5}\times\frac{7}{12}=\frac{28}{45}$.
13. $\frac{7}{11}\times\frac{22}{23}\times\frac{46}{49}=\frac{4}{7}$.

Division of Fractions.

113. RULE.—Reduce mixed or whole numbers to improper fractions, and compound fractions to simple ones, if any such be given. Then multiply the dividend by the *reciprocal* of the divisor, or which is the same, *invert* the divisor, and proceed, in every respect, as in multiplication of fractions.

A complex fraction is reduced to a simple one by dividing the numerator according to the preceding rule.

When a fraction is to be divided by a whole number, the process is the reverse of that in multiplication; or, which is the same, multiply the denominator by the integer, or

divide the numerator by it. The latter mode is to be preferred, when the numerator is a multiple of the divisor.

Example 1. Divide $\frac{5}{18}$ by $\frac{7}{12}$.

$$\tfrac{5}{18} \div \tfrac{7}{12} = \tfrac{5}{18} \times \tfrac{12}{7} = \tfrac{60}{126} = \tfrac{10}{21}.$$

Here, the divisor being inverted, and the fractions being multiplied, we have for the answer $\frac{5}{18} \times \frac{12}{7} = \frac{60}{126} = \frac{10}{21}$.*

2. Divide $5\frac{8}{15}$ by $2\frac{1}{6}$.

Here $5\frac{8}{15} = \frac{83}{15}$, and $2\frac{1}{6} = \frac{13}{6}$; we have, therefore, for the required result, $\frac{83}{15} \times \frac{6}{13} = \frac{83}{5} \times \frac{2}{13} = \frac{166}{65} = 2\frac{36}{65}$.

3. Divide $75\frac{7}{10}$ by 9.

Here, $75\frac{7}{10} = \frac{757}{10}$, and $9 = \frac{9}{1}$; we have, therefore, for the required result, $\frac{757}{10} \times \frac{1}{9} = \frac{757}{90} = 8\frac{37}{90}$. Or a fractional number may be divided by a whole number; thus, 9 is contained 8 times in 75, with the remainder 3. Then $3\frac{7}{10}$, by reduction to an improper fraction, becomes $\frac{37}{10}$, which, by dividing by 9 becomes $\frac{37}{90}$, the remaining part of the quotient. This mode of dividing furnishes an explanation of the rule, in page 43, for finding the true remainder in dividing by the factor of a given divisor.

4. Required the value of the complex fraction $\dfrac{5\frac{1}{8}}{6\frac{3}{5}}$.

Here, $5\frac{1}{8} = \frac{41}{8}$, and $6\frac{3}{5} = \frac{33}{5}$; and the simple fraction required is $\frac{41}{8} \div \frac{33}{5} = \frac{41}{8} \times \frac{5}{33}$, $\frac{205}{264}$.†

* The reason of the operation, and of course the above rule, will appear evident from considering, that $\frac{7}{10}$ divided by 3, for instance, is $\frac{7}{30}$. For if we suppose any whole thing divided first into 10 equal parts, and then into 30 equal parts, the latter being 3 times as many as the former, must each of them be 3 times less than each of the former; and therefore 7 of them must be 3 times less than 7 of the former; or, in other words, seven thirtieths is the third part of seven tenths.

Now, if the divisor were $\frac{3}{4}$, it is plain that the quotient obtained by dividing by $\frac{3}{4}$ must be 4 times the quotient obtained by dividing by 3. We must therefore divide by 3 and multiply the result by 4, which is the same that is done by the inversion of the divisor. From this it appears, that in Division of Fractions, there is in reality, both a division and multiplication. It will also readily appear, that if the divisor be a proper fraction, the quotient will be greater than the dividend.

† It may be useful on some occasions to know, that if the given fractions be reduced to equivalent ones having a common denominator, the quotient of the resulting numerators will be the required quotient. Thus,

Exercises.

1. Divide $4\frac{3}{4}$ by $5\frac{1}{8}$. Ans. $\frac{38}{41}$.
2. Divide $19\frac{1}{20}$ by $2\frac{1}{15}$. Ans. $9\frac{27}{124}$.
3. Divide $\frac{13}{60}$ by $\frac{23}{60}$. Ans. $\frac{13}{23}$.
4. Divide $\frac{7}{9}$ by $\frac{15}{16}$. And. $\frac{112}{135}$.
5. Divide 1 by $\frac{3}{700}$. Ans. $233\frac{1}{3}$.
6. Divide $4\frac{1}{2}$ by 15. Ans. $\frac{3}{10}$.
7. $\frac{7}{8}$ of $\frac{6}{7}\div\frac{4}{5}$ of $\frac{8}{9}$. Ans. $1\frac{7}{128}$.
8. $\frac{2}{3}\div\frac{5}{7}-\frac{5}{8}\div\frac{10}{11}$. Ans. $\frac{59}{240}$.
9. $1\frac{3}{4}\div2\frac{1}{2}+5\frac{1}{2}\div3\frac{1}{8}$. Ans. $2\frac{23}{50}$.
10. What fractional part of 3 is $\frac{2}{3}$ of 4? Ans. $\frac{8}{9}$.
11. What fractional part of 7 is $\frac{3}{7}$ of 5? Ans. $\frac{15}{49}$.
12. A man spent $\frac{1}{4}$ of a legacy in 5 months; $\frac{2}{3}$ of the remainder in 7 months; and then had 95 dollars left. What was the amount of the legacy? Ans. \$380.
13. A man devised $\frac{1}{3}$ of his fortune to his widow; $\frac{2}{3}$ of the remainder to his eldest son; and the rest to his younger. The elder son's share exceeded the younger's by \$750. How much had the widow? Ans. \$3375.

After the full illustration of the multiplication and division of fractions, which has been given in the preceding pages, it appears unnecessary to give their application in the doctrine of ratio and proportion: for the ratio of one fraction to another is found by division of fractions; and a third proportional to three given quantities, either mixed or fractional, is found by dividing the product of the second, and third terms by the first. For instance, to find a fourth proportional to the three given quantities, $17\frac{4}{15}$, $4\frac{1}{7}$, and $3\frac{4}{7}$; or, which amounts to the same thing, to find a fourth proportional to $\frac{259}{15}$, $\frac{29}{7}$, and $\frac{25}{7}$: we have $\frac{259}{15}:\frac{29}{7}::\frac{25}{7}:\frac{10875}{12691}$; which is found by multiplying $\frac{29}{7}$ by $\frac{25}{7}$, and dividing the result by $\frac{259}{15}$: thus $\frac{29}{7}\times\frac{25}{7}\div\frac{259}{15}=\frac{29}{7}\times\frac{25}{7}\times\frac{15}{259}=\frac{10875}{12691}$, the answer.

the fractions in the first example are equivalent to $\frac{10}{36}$ and $\frac{21}{36}$; dividing therefore the numerator 10, by the numerator 21, we have $\frac{10}{21}$, the same quotient as before; and thus may all the exercises in this rule be performed.

Questions.

What is a fraction?

How is a fraction expressed?

What is the number below the line called? what above the line?

What is a proper fraction?

What is an improper fraction?

What does the numerator of a fraction denote?

What does the denominator denote?

If you multiply both terms of a fraction by the same number, does the value of the fraction remain the same?

When can a fraction be reduced to lower terms?

When are numbers said to be prime to each other?

How do you find the greatest common measure of two given numbers?

How is a fraction reduced to its lowest terms?

Repeat the rule for finding the least common multiple of two or more given numbers.

How do you reduce fractions of different denominators to other equivalent ones, having a common denominator?

How is a mixed number reduced to an improper fraction?

How do you find the value of a fraction in the denominations contained in the integer?

How is a given fraction reduced to another of a lower denomination?

Repeat the rule of operation for adding fractional quantities.

Repeat the rule for subtracting fractional quantities.

Repeat the rule for the multiplication of fractions.

Repeat the rule for the division of fractions.

CHAPTER VI.

DECIMAL FRACTIONS.

*On the Nature of Decimal Fractions.**

114. A *decimal fraction*, or a *decimal*, is a fraction whose denominator is 10, or some number produced by the continued multiplication of 10 as factors, such, 100, 1000, &c.

Hence all the rules for the management of fractions in general, are applicable to decimal fractions.

115. In the notation of decimals, the denominator is usually omitted; and, to indicate its value, a point is placed to the left of as many figures of the numerator, as there are ciphers in the denominator. Should there not be a sufficient number of figures in the numerator, as many ciphers are prefixed as supply the deficiency.

Thus, $\frac{7}{10}$, $\frac{9}{100}$, $\frac{3}{10000}$, are decimals; and $\frac{7}{10}$ is written ·7; $\frac{9}{100}$, ·09; $\frac{3}{10000}$, ·0003; $\frac{475}{100}$, 4·75, &c. Hence, conversely:

116. The denominator of a decimal, thus expressed, is the number denoted by a unit with as many ciphers annexed, as there are figures in the given number.

Thus, ·37 is $\frac{37}{100}$; ·004 is $\frac{4}{1000}$; ·00083, $\frac{83}{100000}$, &c.

117. From this notation it is evident, that the figure

* The following articles on *decimal fractions* are merely an adaptation of the general principles of fractional quantities, already delivered, to a particular and important class of fractions. It therefore contains no principles that can be properly called new; but rather a short and easy mode of applying those already given. Every operation on decimal fractions, in fact, may be performed by reducing them to common fractions, as will appear hereafter, and then applying to the results the rules in the preceding articles.

immediately following the decimal point denotes tenths, the next figure hundredths; the third, thousandths, &c.

Thus, since 476 is equivalent to $400+70+6$, the decimal fraction $\cdot476$, or $\frac{476}{1000}$ is equivalent to $\frac{400}{1000}+\frac{70}{1000}+\frac{6}{1000}$, or to $\frac{4}{10}+\frac{7}{100}+\frac{6}{1000}$, by dividing the terms of the first of these frations by 100, and those of the second by 10. In a similar circumstance it would appear, that $\cdot709$ is equivalent to seven tenths, no hundredths, and nine thousandths.

118. Hence, the values of figures in decimals, as well as in whole numbers, are increased in a tenfold ratio by removing them one place towards the left hand, and diminished in the same ratio by removing them one place to the right.

Thus, in the decimal $\cdot004$, by removing the point one place to the right, and consequently the figures of the decimal one place to the left, we have $\cdot04$, which denotes $\frac{4}{100}$, or $\frac{40}{1000}$, and is ten times the given fraction, $\frac{4}{1000}$; but by introducing a cipher after the point, which removes the figures a place to the right, we have $\cdot0004$, or $\frac{4}{10000}$, which is evidently only a tenth part of the given fraction $\frac{4}{1000}$, or $\frac{40}{10000}$. From these principles it will appear evident, that:

119. A decimal is multiplied by 10, if the separating point be removed one place towards the right hand; by 100, if two places; by 1000, if three places, &c.; vacant places, when such occur, being supplied in both cases by ciphers.

Thus, $\cdot7248\times10=7\cdot248$, or $7\frac{248}{1000}$; $6\cdot347\times100=634\cdot7$; $6\cdot3\times1000=6300$. Also, $78\cdot33\div10=7\cdot833$; $\cdot736\div100=\cdot00736$; $7\cdot3\div100=\cdot073$, &c. It appears from the same principles, that:

120. The value of a decimal is not changed by annexing a cipher to the end of it, nor by taking one away, as in each case the significant figures retain the same positions in relation to the separating point.

Thus, $\cdot50=\cdot5=\cdot500$ each being equivalent to one half.*

* From this view of the nature of decimal fractions, it appears, that there is in every respect the closest resemblance between them and whole

Exercises in Notation and Numeration of Decimal Fractions.

Write the following fractions according to the notation of decimal fractions :

1. Three hundred and six ten-thousandths.
2. Three hundred-thousandths.
3. One ten-millionth.
4. One ten-thousandth.
5. Fifty-seven hundred-thousandths.
6. Two hundred ten-millionths.
7. Five hundred and nine-hundredths.

Express the following decimal fractions in words :

1. ·58	9. 345·689
2. ·106	10. 34·5689
3. ·007	11. 3·45689
4. ·0007	12. ·345689
5. ·00007	13. ·00004
6. ·000007	14. ·000041
7. ·0000001	15. ·00000011
8. 3456·89	

Reduction of Decimal Fractions.

PROBLEM I.—*To reduce a common fraction to a decimal.*

121. RULE.—Let the numerator, with a cipher annexed, be divided by the denominator, and to the significant figure or cipher placed in the quotient, prefix a point. Then if there be a remainder, annex to it more ciphers, and continue the division till nothing remains, or till the result consist of as many figures as may be thought necessary.

numbers; and hence all operations on decimals are performed in exactly the same manner as those on whole numbers: due attention being paid to the position of the separating point. This last circumstance, indeed, demands the utmost care, as *the point* is the characteristic of the decimal; and from what precedes, it is evident how much depends on the proper position.

A given quantity may be reduced to the decimal of another given quantity of the same kind, considered as an integer, by first reducing it to a common fraction by problem VII. page 132, and then reducing the fraction thus found to a decimal.

Example 1. Reduce $\frac{3}{512}$ to a decimal.

Here, 3 by the addition of one cipher, becomes 30, in which 512 is not contained; and therefore a cipher is put in the quotient, and a point prefixed. By the annexing of another cipher, 30 becomes 300, in which the divisor not being contained, another cipher is put in the quotient. After this the division proceeds in the usual way, a cipher being added each time, and the decimal is found to be ·005859375, or $\frac{5859375}{1000000000}$, which by reduction to its lowest terms, would become $\frac{3}{512}$, the given fraction; and thus the work is proved to be correct.*

512)3000(·005859375
4400
3040
4800
1920
3840
2560
...

2. Reduce $\frac{5}{12}$ to a decimal.

In this example, each remainder after the first two is 8, and hence the same figure must be repeated perpetually.

12)50000
·41666, &c.

3. Reduce $\frac{6}{11}$ to a decimal.

Here, after two figures have been obtained in the quotients, the same series of remainders, and consequently the same series of figures in the quotient,

11)60000
·5454, &c.

* With respect to *the reason of the rule*, the given fraction by annexing to each of its terms nine ciphers, (the number that was annexed in the preceding operation,) becomes $\frac{3000000000}{512000000000}$; and this, by dividing each term by 512, becomes $\frac{5859375}{1000000000}$, which in the notation of decimals, is ·005859375, the same as before. Hence it is obvious, that the preceding reduction is in reality the multiplication of each term of the given fraction by 1000000000, and the division of each of the results by 512; and it is evident from the principles established in the first section of the preceding chapter, that the result must be equivalent to the given fraction.

must recur; and therefore, were the work pursued, the first two figures of the decimal would recur without end.

4. Reduce 4 cents to the decimal of a dollar.

Here, 4 by the addition of one cipher, becomes 40, in which 100 is not contained; and therefore a cipher is put in the quotient, and a point prefixed. By the annexing of another cipher, 40 become 400, in which the divisor is contained 4 times, without remainder, and the decimal is found to be ·04.

$$100)400$$
$$\overline{\cdot 04} \text{ Ans.}$$

5. Reduce 13*s*. 8*d*. to the decimal of a pound.

Here, 13*s*. 8*d*.=164 pence=£$\frac{164}{240}$=£$\frac{41}{60}$; and £$\frac{41}{60}$, reduced to a decimal in the manner already shown, becomes £·6833, &c. The same result might be obtained by writing, as in the margin, 13 shillings below 8 pence; then dividing by 12 and by 20, we find 13*s*. 8*d*. equivalent to 13·6666, &c. shillings, or to £·6833, &c. the same as before.

$$12)8.0000$$
$$20)13.6666, \&c.$$
$$£ \cdot 6833, \&c.$$

122. A decimal which cannot be exactly expressed, but which may be continued to an unlimited number of figures, is called an *interminate decimal*, to distinguish it from others, which in respect of it, are called *terminate*.

123. When a decimal is expressed either by the continual repetition of the same figure, or of the number expressed by two or more figures, it is called a *periodical* or *circulating decimal;* and the figure, or number, so repeated is called the *period*.

Thus the decimals in the second and third examples are periodical, the period in the one consisting of one figure, and that in the other of two. When only one figure is repeated, the application of the terms *period* and *periodical*, though convenient, and perhaps on the whole, is scarcely correct, as each of the terms suggests the idea of more figures than one. On this account, some writers term such decimals, *repeaters* or *repeating decimals*.

124. A periodical decimal is said to be *mixed*, if it

consist of one or more figures prefixed to a periodical part; others are called *pure*.

For the sake of brevity, in writing decimals of this kind, it will often be sufficient to write the period but once, and to denote its continuation by putting a trail, or accent over the first figure of the period, and another over the last, or one over the repeating figure, if there be but one figure in the period.

Thus, ·47333, &c. may be expressed by ·473′, and ·5637637, &c. by ·56′37′.

The following considerations will tend to explain still farther the nature of these fractions. By the preceding rule we find the following results: $\frac{1}{9}=\cdot1111$, &c.; $\frac{1}{99}=\cdot010101$, &c.; $\frac{1}{999}=\cdot001001$, &c.; $\frac{1}{9999}=\cdot00010001$, &c. These decimals are all periodical; and if any of them be multiplied by a whole number, the result will be periodical. Thus, if we multiply the third by 128, we find $\frac{128}{999}=\cdot128128$, &c.; if by 998, we get $\frac{998}{999}=\cdot998998$, &c. Here we see that the value of the periodical fraction is the common fraction, whose denominator is the number expressed by as many nines as there are figures in the period, and whose numerator is the period itself: and the same may be shown in every case. Hence, we have the following rule: *To find the value of a pure periodical decimal, take the period for numerator, and as many nines as there are figures in the period, for denominator.*

Thus, ·8181, &c.$=\frac{81}{99}=\frac{9}{11}$; ·2′97′$=\frac{297}{999}=\frac{11}{37}$; ·3′$=\frac{3}{9}=\frac{1}{3}$; ·999, &c.$=\frac{9}{9}=1$, &c.

If the decimal be mixed, its value may be easily found by the same principle. Thus, if it were required to find the common fraction which would produce the decimal, ·12436436, &c. by multiplying by 100 we should find 12·4′36′, or $12\frac{436}{999}$. Dividing this by 100, we obtain, for the value of the proposed fraction, $\frac{12}{100}+\frac{436}{99900}$, or by reduction to a common denominator, $\frac{12\times999}{99900}+\frac{436}{99900}=\frac{12424}{99900}=\frac{3106}{24975}$. From this we may readily derive a rule which will be easier in practice. For in the last result, since 12 is multiplied into 999, or into 1000—1, the product is 12000—12; to which if the other numerator be added, we have for the required numerator 12436—12; and the denominator remains the same as before. From a due consideration of this result

we may derive the following rule: *To find the value of a mixed periodical decimal, from the number expressed by the finite part with the period annexed, subtract the finite part for the numerator; and, for the denominator, to as many nines as there are figures in the period, annex as many ciphers as there are figures in the finite part.*

Thus to find the value of 83', from 83 take 8, and there will remain 75, and the required fraction is $\frac{75}{90}$, or $\frac{5}{6}$. Again, if ·5416' be given, we have 5416—541, or 4875 for numerator, and the fraction is $\frac{4875}{9000}$, or $\frac{13}{24}$. In like manner, to find the value of ·263'01', we have for numerator 26301 —26=26275, and for denominator 99900. Hence the required fraction is $\frac{26275}{99900}$, or $\frac{1051}{3996}$.*

Exercises.—Reduce the following fractions to decimals:

1. $\frac{7}{16}$	Ans.	·4375	6. $\frac{1}{9}$	Ans.	·1'
2. $\frac{3}{32}$		·09375	7. $\frac{4}{909}$		·0'044'
3. $\frac{11}{40}$		·275	8. $\frac{4}{99}$		·0'4'
4. $\frac{1}{13}$		·0'76923'	9. $\frac{1}{44}$		·022'7'
5. $\frac{82}{12}$		·683'	10. $\frac{4}{9009}$		·0'0044'

11. Reduce $\frac{1}{81}$ to a decimal. Ans. ·0'12345679'.
12. Reduce $\frac{37}{79}$ to a decimal. Ans. ·4'683544303797'.
13. Reduce 10*s*. 9*d*. to the decimal of £1. Ans. ·5375.
14. Reduce 6 cents, 3 mills to the decimal of $1. Ans. ·063.
15. Reduce 3 acres, 11 poles to the decimal of an acre. Ans. ·8187.
16. Reduce 2*qrs*. 8*lbs*. to the decimal of a *cwt*. Ans. ·5'71428'.

* By the method thus shown, interminate decimals may be reduced to common fractions, and subjected to the rules for managing such quantities. Unless when complete precision is required, however, this is not necessary: and indeed in all useful cases their values, instead of being found with entire accuracy, are to be *approximated*, by carrying the decimals out to as many figures as may be necessary in a particular case. In thus approximating to the value of a decimal, *if it be carried to two places, the error will be less than a hundredth part of the integer; if to three places, less than a thousandth; if to four places, less than a ten thousandth*, &c. Thus, if the decimal ·2'7', be carried only to ·27 the error would be less than $\frac{1}{100}$, since the true value is more than $\frac{27}{100}$, and less than $\frac{28}{100}$; if to the three figures ·272, the error would be less than $\frac{1}{1000}$, since the true value is greater than $\frac{272}{1000}$, and less than $\frac{273}{1000}$, &c.

17. Reduce 37 poles to the decimal of a mile.
Ans. ·115625.

18. Reduce 3 hours, 30 minutes to the decimal of a day.
Ans. ·14583'.

19. Reduce 15 minutes, 30 seconds to the decimal of an hour.
Ans. ·2583'.

20. Reduce 3*cwt.* 1*qr.* 7*lbs.* to the decimal of a ton.
Ans. ·165625'.

21. If the diameter of a circle be 1, the circumference is $3\frac{1}{7}$ nearly, or $3\frac{16}{113}$ more nearly. Express each of these decimally.* Ans. 3·14'2857', and 3·1415929, &c.

22. If the circumference of a circle be 1, and the diameter is $\frac{7}{22}$ nearly, or $\frac{113}{355}$ more nearly. Express each of these decimally. Ans. ·3'18', and ·308309859, &c.

23. An American acre is, $\frac{121}{196}$ of an Irish acre: required the equivalent decimal. Ans. ·6173469387755l, &c.

24. The pound troy is $\frac{144}{175}$ of a pound avoirdupois: required the equivalent decimal. Ans. ·822'85714'.

PROBLEM II.—*To find the value of a given decimal in the parts contained in the integer.*

125. RULE.—Multiply the given decimal by the numbers which, if it were an integer, would reduce it to the lower denominations contained in it, and, after each multiplication, point off for decimals as many figures toward the right hand as there were figures in the given decimal. The figures remaining on the left will express the required value.

Example 5. Required the value of ·3945 of a day in hours, &c.

* The circumference true to 20 places, is 3·14159265358979323846.

Here the given decimal is multiplied by 24, the number of hours in a day, and four figures being cut off towards the right hand, it appears that ·3945 day is equal to $9\frac{4680}{10000}$ or $9\frac{468}{1000}$ hours. The decimal ·4680 is then multiplied by 60, and four figures being cut off, there results 28·0800 or 28·08 minutes. By continuing the process, the value of the given decimal is found to be 9 hours, 28 minutes, 4 seconds, 48 thirds.*

```
  ·3945 day.
     24
  ------
  15780
  7890
  ------
  9·4680 hours.
     60
  ------
 28·0800 min.
     60
  ------
  4·8000 sec.
     60
  ------
 48·0000 thirds.
  ------
```

6. Required the value of ·805′ of a yard in long measure.

This, and similar exercises, may be wrought either by converting the proposed decimal into a common fraction, in the way shown in page 150, or more easily by employing an approximate process, as in the margin. Here we carry 1 to the product of 3 and 5, because had the decimal been continued farther, it is evident 1 must have been carried from the preceding product. For a similar reason, 7 is carried to the product of 12 and 6. The result is found to be 2 feet, 4·9′ inches. (*see page* 149.) Another 5 was added to the given decimal, that the result might be more distinct and certain.

```
 ·805′ of a yard
    3
 ------
 2·4166
    12
 ------
 4·9999 inches.
 ------
```

Exercises.—Required the values of the following decimals.

1. Required the value of ·0675 of a *cwt.* Ans. $7\frac{14}{25}$*lbs.*
2. - - - - - - - - ·4625 of a ton. Ans. 9*cwt.* 1*qr.*
3. - - - - - - - - - £·5937. Ans. 11*s.* $10\frac{61}{125}$*d.*

* With respect to the reason of this process, it is only necessary to observe, that it is exactly the same as finding the value of $\frac{3945}{10000}$ day by problem VII. page 132, the pointing off of decimals serving the purpose of dividing by the denominator.

4. Required the value of ·8845 of an acre.
Ans. 3 *roods*, $21\frac{13}{25}$*p*.

5. - - - - - - - - - ·00213 of a day.
Ans. 3 *min.* $4\frac{4}{125}$ *sec.*

6. - - - - - - - - - 2′85714′ of a *cwt.*
Ans. 1*qr.* 4*lbs.*

7. - - - - - - - - - ·113′6′ of an American mile.
Ans. 36*p.* 2*yds.*

8. - - - - - - - - - ·2385 of a degree.
Ans. 14′ 18″ 36‴.

9. - - - - - - - - - ·47916′ of a *lb.* troy.
Ans. 5*oz.* 15*dwts.*

10. - - - - - - - - - ·4375 of a shilling.
Ans. $5\frac{1}{4}$*d.*

11. - - - - - - - - - ·09375 of an acre.
Ans. 15 *poles.*

12. - - - - - - - - - ·4′ foot, *long measure.*
Ans. 5 *inches*, 4 *lines.*

13. What is the value of ·67 of a league.
Ans. 2*m.* 3*p.* 1*yd.* $3\frac{3}{5}$*in.*

14. What is the value of ·61 of a tun of wine.
Ans 2*hhd.* 27*gal.* 2*qt.* 1*pt.*

Addition of Decimal Fractions.

126. Rule.—Arrange the given numbers so that the separating points may be all in the same column. Find the sum as in addition of whole numbers. Point off as many of the decimals, as there are in the given number which contains the most.

If any of the given numbers contain periodical decimals, let these be carried out to as many places as there are in the longest line of the finite decimals; or, if greater accuracy be required, let them be carried as far as may be judged necessary.*

* In the application of decimals to practical purposes, it is generally known from the nature of the case under consideration, to how many places it is necessary, that the result may be true. *When a result is thus required to be true to an assigned number of places of decimals, it is proposed to carry the decimals, which consist of more places, to at least one place beyond the assigned number, and to reject the last figure.* In this case, it is proper to observe, that *when a decimal is not carried out to its full length, the last figure of the part retained, should be increased by a unit, if the succeeding figure be 5 or greater than 5.*

Example 1. Add together 81·4632, 9·75, and 47·388.

Here, the numbers are arranged as in the margin, and added as in addition of whole numbers. The reason of the arrangement and operation is quite manifest, those figures being added together which are of the same local value.

```
 81·4632
  9·75
 47·388
————————
138·6012, sum.
```

2. Add together 3·7′3′, ·873, 51·7′, 108·2, and 73·463128; so that the result may have four places of decimals true.

In this example, the first, third, and fifth numbers are carried to five places each, and the last figure of the third is made 8, because the next figure would be 7. In like manner the fifth figure of the last line is made 3, because the succeeding figure is 8.*

```
  3·73737
   ·873
 51·77778
108·2
 73 46313
————————
238·0513, sum.
```

Exercises.—1. Add together 1·83, 5·674, ·3125, 18·3, 100, 38·62, 4·3957, and ·5. Ans. 169.6222.

2. Required the sum of 93·617843, 7·836, 12·25, ·71375, 4·391, 7·839, 3·7674285, and ·8693. Ans. 131·2843215.

3. Required the sum of ·7354, ·7354′, ·735′4′, ·7′354′, ·07354, and ·0735′4′. Ans. 3·088857′991′.

4. Required the sum, true to five places, of the numbers given in exercises 5th, 6th, and 13th of Addition of Fractions, the several fractions being previously reduced to decimals.

Ans. 6·0078125, 3·907′14285′, and 1·561011′90476.

Subtraction of Decimal Fractions.

127. Rule.—Set the less number so that each figure in it may stand below a figure of the same local value in the greater. Then find the difference as in subtraction of whole numbers, and place the separating point as in addition of decimals.

* The reason of this is evident, since 30 is nearer 28 than 20 is, and 30, by the rejection of the last figures, becomes 3. In the addition, the sum of the last column is 18, from which 2 is carried, because 18 is nearer 20 than 10. The correct sum, found by carrying the decimals farther, is 238·0512795′1′, which by retaining only four figures of the decimal, and increasing the last of them by a unit, because it is followed by 79, &c. becomes 238·0513, the same as before.

Example 1. From 3·5′4′ take 1·34265.

Here, the greater number is extended, and the remainder is found to be 2·202804′5′.	3·5454545
	1·34265
	2·202804′5′ diff.

2. Required the difference of 8·6 and 2·7′

Here, the less number is carried to four places, that the true answer may be discovered with greater certainty. In the subtraction, ciphers are conceived to be annexed to the greater	From 8·6
	Take 2·7777
	Rem. 5·8222

number, and 1 is carried to the repeating figure first used, because this must have been done, had the less number been carried one place farther. The answer is found to be 5·82′.

Exercises.—1. Required the difference of 3·468 and 1·2591. Ans. 2·2089.

2. Required the difference of 6·45 and 1·34′5′. Ans. 5·104′5′.

3. Required the difference of 13·6′ and 4·345. Ans. 9·3216′.

4. Required the difference of ·682 and ·09647. Ans. ·58553.

5. Required the difference of 5·83 and 4·1′7′. Ans. 1·658′2′.

6. Required the difference of $17\frac{3}{8}$ and $7\frac{3}{7}$.* Ans. 9·94642857.

7. Required the difference of $7\frac{5}{72}$ and $4\frac{7}{65}$. Ans. 2·9617521.

8. Required the difference of $15\frac{1}{17}$ and $13\frac{6}{25}$. Ans. 1·8188235.

Multiplication of Decimal Fractions.

128. Rule.—Multiply the factors as in multiplication of whole numbers, and point off in the product as many of the decimals as there are in both factors, sup-

* In this exercise and the next two, the given fractions are to be reduced to decimals, and the difference taken according to the rule.

plying the deficiency, when any occurs, by prefixing ciphers.

Example 1. Multiply ·582 by 66·3.

Here, because there are three places of decimals in the one factor, and one in the other, there must be four places of decimals in the product.*

$$\begin{array}{r} \cdot582 \\ 66\cdot3 \\ \hline 1746 \\ 3492 \\ 3492 \\ \hline 38\cdot5866 \end{array}$$ product.

2. Multiply ·13 by ·7.

Here, a cipher must be prefixed to the product 91, as there are two places of decimals in one factor, and one in the other.

$$\begin{array}{r} \cdot13 \\ \cdot7 \\ \hline \cdot091 \end{array}$$

Exercises.	*Answers.*
1. ·78 × ·42	= ·3276
2. 7·8 × 4·2	= 32·76
3. 7·49 × 63·1	= 472·619
4. ·1 × ·1 × ·1 × ·1 × ·1	= ·00001
5. ·08 × ·036	= ·00288
6. ·144 × ·144	= ·020736
7. 36·48 × ·475	= 17·328
8. 13·825 × 5·128	= 170·8946
9. ·31 × ·32	= ·0992
10. 62·38 × 7	= 436·66

Ex. 11. Required the product of 7·24651 and 81·4632.

Ans. 590·323893432.†

* *The reason of this rule* will be understood from considering, that when the denominators are supplied, the first factor becomes $\frac{582}{1000}$, and the second $66\frac{3}{10}$ or $\frac{663}{10}$, which, by the rule for the multiplication of fractions, page 112, give for product $\frac{582 \times 663}{10000}$; whence it appears, that the product of 582 and 663 must be divided by 10000, which is effected by cutting off four figures. It is evident that the divisor must contain as many ciphers as there are in both denominators; that is, as there are decimal figures in both factors.

† When the number of decimal figures is great, or the factors numerous, the decimal figures resulting from the application of the preceding

Division of Decimal Fractions.

129. Rule.—If the divisor and dividend do not contain the same number of places of decimals, supply the deficiency by annexing ciphers, or in a periodical decimal, the next figure of the period. Then rejecting the separating points, divide as in whole numbers, and the quotient will be a whole number. If there be a remainder after all the figures of the dividend have been employed, ciphers or periodical figures may be annexed, till there be no remainder, or till as many figures be found as may be judged necessary.

rule, are, in many cases, unnecessarily and inconveniently numerous. The following approximative rule will be found extremely useful in such cases.

Rule.—*Count off, after the separating point in the multiplicand, (annexing ciphers if requisite,) as many figures of decimals as it is necessary to have in the product. Below the last of these, write the unit figure of the multiplier, and write its other figures in reversed order.*

Then multiply by each figure of the multiplier thus inverted, neglecting all the figures of the multiplicand to the right of that figure, except to find what is to be carried; and let all the partial products be so arranged, that their right hand figures may stand in the same column. Lastly, from the sum of these partial products, cut the assigned number of decimal places.

In carrying from the rejected figures, we should always take what is *nearest* the truth, whether it be too great or too small.

Let us take, for instance, the above example, in order to illustrate this rule. Multiply 7·24651 by 81·4632, so that there may be only three places of decimals in the product.

Here 1, the unit figure of the multiplier, is written below 6, the third decimal figure of the multiplicand; 8, the figure which precedes 1, is written after it; 4, the figure which follows it, is set before it, &c. We then say, 8 times 5=40, and 1, carried from 8 times 1, =41; 1 is then set down, and 4 carried, and the rest of the multiplication by 8 proceeds in the usual way. Then, in multiplying 7·246 by 1, we add 1 to the product for 51, because 51 is nearer 100 than 0, and therefore it is nearer the truth to carry 1 than 0. In multiplying 7·24 by 4, three is carried for the product that would have resulted from the rejected figures: for going two places back, we have 4 times 5=20; 4 times 6=24, and 2=26, which being nearer 30 than 20, we carry 3. For a similar reason, in multiplying 7·2 by 6, we carry 3 from the rejected figures; and thus we proceed in similar cases.

	7·24651
	2364·18
	579721—
	7247—
	2899—
	435—
	22—
	1+
prod.	590·325

P

If, after the rejection of the separating points, the divisor be greater than the dividend, the quotient will contain no whole number, and the work will proceed according to the rule for Problem I. in Reduction of Decimals.

When the divisor is large, the work will be shortened, if, instead of annexing a cipher or periodical figure to each remainder, a figure be cut off from the divisor. In this case, each product is to be increased by *carrying* from the product of the figure last cut off, and of the figure last placed in the quotient.

Ex. 1. Divide 1346·5 by 43·68.

Here, by annexing a cipher to the dividend, and rejecting the points, we have for divisor 4368, and for dividend 134650. Hence, dividing in the common way, we find 30 for the integral part, and annexing ciphers to the remainders, and continuing the operation, we get ·826465, &c.; and the entire answer is 30·826465, &c. The work is left for the learner to perform.*

2. Divide ·1342 by 67·1.

Here, by annexing three figures to the divisor and rejecting the points, we get for divisor 671000, and for dividend 1342: then the dividend being the greater, the quotient will contain no integral part; and the annexing of a cipher to the dividend gives one cipher for the quotient: the annexing of a second gives another cipher; but the addition of a third gives 2. Hence the quotient is ·002.†

* With respect to the *reason* of the operation, the value of 1346·5 is not changed by the annexing of a cipher; and the removal of the points merely multiplies each of the given numbers by 100. (see p. 119.) It is therefore evident, that the value of the quotient will not be affected, since, while the dividend is multiplied by 100, the divisor is increased in the same ratio. We might also consider the dividend as the numerator, and the divisor as the denominator of a fraction, and then the reason of the process would depend on the principle established in § 88. The reason of removing the points is to make the dividend and divisor whole numbers, and thus to render the operation as much as possible the same as in division of whole numbers.

† When the number of places of decimals in the divisor is not greater than the dividend, the number of figures of decimals in the quotient is to be equal to the difference between the number of places in the divisor and dividend, as is evident from Multiplication of Decimals; and in this way the number of decimal figures in the quotient is often very easily determined.

3. Divide 1·7154 by 1·5.

```
1·5)1·7154(1·1436
    1 5
    ---
     21
     15
     --
      65
      60
      --
       54
       45
       --
        90
        90
        --
```

In this example there are five decimals in the dividend, (0 being added to the remainder,) and but one decimal in the divisor; there must therefore be four decimals in the quotient, which is 1·1436.

4. Divide 2·3748 by 1·4736.

```
14736)23748(1·611
  •••  14736
       -----
        9012
        8842
        ----
         170
         147
         ---
          23
          15
          --
           8
           -
```

```
14736)23748(1·611
      14736
      -----
       9012|0
       8841|6
       ----|-
        170|40
        147|36
        ---|--
         23|040
         14|736
         --|---
          8|304
          -----
```

In this example the numbers being prepared according to the rule, and the first figure of the quotient being found, instead of adding a cipher to the remainder 9012, we omit the last figure of the divisor, to denote which a point is placed below it. Then 6 being put in the quotient, we multiply by 6, the figure cut off, by it, and without setting any thing down, we carry 4, because the product, 36, is nearer 40 than 30. After that, 3 is cut off in like manner, and then 7. The quotient is found to be 1·611, or more nearly 1·612, because the remainder 8 is rather more than half of 14 The annexed operation at full length, will explain the reason of

the contracted process, the vertical line cutting off the contracted part.

Exercises.—1. Divide 73·64 by ·43'2'.*

Ans. 170·3355.

2. Divide 47·58 by 26·175. Ans. 1·81776504.
3. Divide ·3412 by 8·4736. Ans. ·040266239.
4. Divide ·5'8' by 77·482. Ans. ·00756122.
5. Divide ·6'1' by 13·543516. Ans. ·04549495.
6. Divide 1 by 10·473654. Ans. ·09547766.
7. Divide ·0'9' by ·2'30769'. Ans. ·3'9'.
8. Divide 52·7'3' by 52·734567. Ans. 1·00005322.
9. Divide ·079085 by ·83497. Ans. ·094716.
10. Divide 24877·4, the number of miles in the circumference of the Earth, by 360, the number of degrees into which a great circle is divided. Ans. 69·1039.
11. When the diameter of a circle is 1, the circumference is 3·1416 nearly; what is the diameter of the Earth, allowing its circumference to be 24877·4 miles?

Ans. 7918·7 nearly.

Questions.

What are decimal fractions?

How are decimals written?

What does the figure following the decimal point denote? what the next, &c.?

How do the values of figures in decimals increase and decrease?

If a cipher be annexed to the end of a decimal, or if one be taken away, is the value of the decimal changed?

Repeat the rule for reducing a common fraction to a decimal.

Repeat the rule for finding the value of a given decimal in the parts contained in the integer.

How are the numbers arranged for performing the addition of decimals?

Repeat the rule of operation.

Repeat the rule for the subtraction of decimals.

Repeat the rule for the multiplication of decimals.

Repeat the rule for the division of decimals.

* Here it would be necessary to extend the divisor to eight places of decimals, and the dividend to the same number.

CHAPTER VII.

Practical Application of the Rule of Proportion.

130. The *rule of proportion* is divided into *simple* and *compound.*

Simple proportion is the equality of the ratio of two quantities, to that of two other quantities. (*see* § 65.)

Compound proportion is the equality of the ratio of two quantities to another ratio, the antecedent and consequent of which are respectively the products of the antecedents and consequents of two or more ratios. (*see* § 82, *chap.* IV.)

Simple Proportion.

131. The object of that part of simple proportion which is usually taught in courses of arithmetic, is to find the number which has the same ratio to one of three given numbers, that there is between the other two; or to find a fourth proportional to three given numbers.

Example 1. If I can purchase 4 yards of cloth for $35·50, what quantity ought I to get, at the same rate, for $106·50?

This can be ascertained by simple proportion: for the quantities purchased at a given rate must be directly as the prices paid; therefore 4 yards the quantity purchased for $35·50 must be less than the quantity purchased for $106·50, in the same ratio in which the former sum of money is less than the latter, or in the ratio 3550 cents to 10650 cents, or of the abstract numbers 3550 : 10650. Therefore, 3550 : 10650 : : 4 yards to the quantity sought; which fourth proportional is found (§ 68) by taking the product of the second and third terms and dividing that product by the first: thus,

$$\frac{10650 \times 4}{3550} = \frac{42600}{3550} = 12 \text{ yards.}$$

2. If I pay $106·50 for 12 yards of cloth, what must I pay, at the same rate, for 4 yards?

Here the thing sought being a sum of money, the given sum of money, $106·50 must be the third term of the analogy. And as the answer must be a less sum of money, the two given quantities of cloth must be stated in a greater inequality; that is, as 12 to 4. So that, as 12 : 4 :: 10650 cents to the sum sought. Therefore $10650 \times 4 \div 12 = 3550$ cents, or $35·50, the answer. This question is formed from the last, and the result obtained is sufficient to prove the accuracy of the work in both solutions.

3. If a mason can build a wall in 6 days, working 7 hours a day, how many hours a day must he work in order to build it in 5 days?

It is plain that he must work a greater number of hours each day, and therefore the fourth term of the analogy must be greater than the third term, 7 hours; and hence the first two terms must be stated in a ratio of less inequality; thus 5 : 6 :: 7 hours to the number of hours sought. The answer, therefore is $\frac{42}{5}$, or $8\frac{2}{5}$ hours; that is, 8 hours and 24 minutes. The truth of this may be proved by forming another question in which this answer shall be one of the given terms, and any one of the former given terms shall be the term sought. Thus:

4. If a mason, working 8 hours and 24 minutes a day, build a wall in 5 days, how many hours a day must he work in order to build it in 6 days?

Here it is plain that he must work a less number of hours each day; and therefore the fourth term of the analogy must be less than the third term, $8\frac{2}{5}$ hours: and hence the first two terms must be stated in a ratio of greater inequality, that is, as 6 : 5 :: $8\frac{2}{5}$ hours to the number of hours sought. The answer, therefore, is $8\frac{2}{5} \times 5 \div 6$, or $\frac{42}{6} = 7$ hours.*

* We may, in like manner, form two other questions from Ex. 3.—Thus: if a mason working 8 hours and 24 minutes a day, build a wall in 5 days, in how many days shall he build it, working 7 hours a day? Or, 2dly, if he build it in 6 days, working 7 hours a day, in how many days shall he build it, working each day 8 hours and 24 minutes? And thus whenever a question has been solved by the rule of proportion, the student may be profitably exercised in forming three other questions adapted to prove the truth of his answer, since we can find any one of the four terms of an analogy from having given the three others.

It is proper to observe that the first two examples are performed by what is usually called *Direct Proportion*, and the last two examples by *Inverse Proportion*; or, in other words, by what is commonly, though improperly, called, "The Rule of Three *direct* and *inverse*." This distinction is perfectly useless; and, like all useless distinctions, it is calculated only to perplex the learner, and to render a simple subject complicated. The preceding four examples may serve to illustrate the following general rule for solving all questions in simple proportion, whether *direct* or *inverse*.

Three numbers being given, to find a fourth Proportional.

132. Rule.—Arrange the three given terms in the same line, in succession, placing the one which is of the same kind with the required term the third in order; and if, by the nature of the question, the required term is to be greater than the third term, put the greater of the other two terms in the second place; otherwise put the less in that place. Then, if the first two terms be not of the same simple denomination, reduce them to the same denomination, usually the lowest mentioned in either. Find the product of the second and third terms, and divide it by the first. The quotient is the fourth proportional in the same denomination as the third term, and may be reduced to a higher denomination if necessary.*

* In the method of arranging the terms which is delivered in almost all the books on Arithmetic, and which is very generally employed in practice, the term which is of the same kind with the answer is put in the second place. This method is entirely subversive of the principles of proportion, and is calculated to prevent the learner from acquiring just views of this subject, as in it a ratio is, in many cases, instituted between quantities entirely different in kind. The method above explained (taken from *Thomson's Arithmetic*, and which has also been adopted by *Walker* and some other late writers) is founded on principles strictly mathematical; and besides being very simple and easy in practice, it possesses the advantage of training the mind to accurate thinking, and of preparing it for the subsequent study of Mathematics. It also precludes the necessity of what is generally called *the Rule of Three Inverse;* as all the questions usually solved under that head, may be solved by the rule above delivered.

When the first and second terms are in the same denomination, they are evidently in the same ratio as the numbers which express them; and therefore they are to be reduced to the same denomination, if they be not so already.

Ex. 1. If 12 yards of cloth cost 33 dollars, what would 8 yards cost at the same rate?

The answer to this question must evidently be in money, and therefore \$33, the money given in the question, must occupy the third place: and, as 8 yards will cost less than 12 yards, 8 yards, the less of these two terms, must be put in the second place. Hence, the terms will be arranged, and the operation performed, as in the margin; and it appears that, at the rate specified in the question, 8 yards would cost \$ 22.*

```
As 12 : 8 :: 33 :
              8
           ----
         12)264
           ----
           $ 22
```

2. How many men would perform in 168 days a piece of work which 108 men can perform in 266 days?

Here, it is plain that the answer to this question must be men; and therefore the given number of men must be put in the third place; and as the answer must be a greater number of men, in order to perform the work in less time than 266 days, we must put 266, the greater of the other two terms, in the second place: hence the terms will be arranged and the operation performed, as in the margin; and the answer is found to be 171 men.†

```
As 168 : 266 :: 108 :
                266
                ---
                648
               648
              216
              -----
        168)28728(171
            168
            ----
            1192
            1176
            ----
             168
             168
             ---
```

* In this example, it is evident that as often as 8 yards are contained in 12 yards, so often would the answer be contained in \$ 33. Hence the quantities are proportionals, and the reason of the process is evident from what goes before.

The reason of the operation will also be plain if it be considered, that the price of one yard would be found by dividing \$ 33 by 12, and

† It is obvious that the terms must be thus arranged: since 266×108 would be the number of days in which the work would be performed by

Exercises.—1. If 57*cwt.* of sugar cost $ 432, what would 95*cwt.* cost at the same rate? Ans. $ 720.

2. If 385*yds.* of linen cost $ 315, how much might be bought for $ 90? Ans. 110*yds.*

3. If the yearly rent of a farm containing 182 acres be $ 273, what is the rent of a part of it containing 43 acres? Ans. $ 64·50.

4. If 275 reams of paper cost $ 990, what would 990 reams cost? Ans. $ 3564.

5. If 96 men reap 40 acres of grain in a week, how many men would reap 65 acres in the same time? Ans. 156.

6. If 84 sheep can be grazed in a field for 12 days, how long might 112 sheep have been grazed equally well in the same field? Ans. 9 days.

7. If the shilling loaf weighs 36 ounces when flour is $ 4 per barrel, how much must it weigh when flour is $ 6 per barrel? Ans. 24*oz.*

8. If a person lent me $ 270 for 8 months; in return for his kindness how much ought I to lend him for 18 months? Ans. $120.

9. How many men must be employed to finish a canal in 12 days, which 5 men could perform in 36 days? Ans. 15.

10. If a person travel 12 hours a day, and finish his journey in three weeks, how long should the same journey take him if he travelled only 9 hours a day? Ans. 4 weeks.

11. If 24 pioneers can make a trench in 12 days, what length of time would the same work employ 9 men? Ans. 32 days.

12. Suppose 50 men build a house in 60 days, how many men would build the same in 100 days? Ans. 30.

13. If a besieged garrison have 4 months' provisions, at the rate of 18 ounces per day for each man, how long will

that the price of eight yards would be found by multiplying the quotient by 8; and the result will evidently be the same that would be obtained by multiplying $ 33 by 8, and dividing the product by 12. The method according to the rule is in general preferable, however, as it has the advantage of freeing the operation as much as possible from fractional quantities.

one man; and if this be divided by 168, the quotient must be the number of days required; consequently, 266 and 108 must be made the second and third terms that their product may be taken in the operation. This question, as well as many others in this article, belongs to *Inverse Proportion*.

...e able to hold out if each man is allowed only 12 ...s per day? Ans. 6 months.

...am. 3. If 12*cwt.* of wheat cost £6 10*s.* 8*d.* how much ...t be bought for £11 8*s.* 8*d.*?

	£	*s.*	*d.*		£	*s.*	*d.*		*cwt.*
As	6	10	8	:	11	8	8	::	12
	20				20				
	130				228				
	12				12				
	1568				2744				
					12				

1568)32928(21*cwt.* answer.
3136
1568
1568

In this example, according to the second part of the rule, the first and second terms are reduced to the same denomination, (pence,) and the rest of the work proceeds as before.

Exercises.—14. If 148 gallons of wine cost £119 10*s.*, how much may be bought for £89 12*s.* 6*d.*? Ans. 111 gallons.

15. What is the rent of 21 acres, 3 roods, 20 poles of ground, if the rent of 36 acres, 3 roods be $ 42? Ans. $ 25.

16. If $ 102 pay for 10*cwt.* 2*qrs.* 14*lb.* of sugar, what would 4*cwt.* 1*qr.* 14*lb.* cost at the same rate? Ans. $ 42.

17. If a person walk 17 miles in 5 hours, 12 minutes, 31 seconds, how far would he walk at the same rate in 3 hours, 40 minutes, 36 seconds? Ans. 12 miles.

18. If the fare of a coach for 88 miles be £1 9*s.* 4*d.* for what distance would £2 3*s.* 4*d.* pay at the same rate? Ans. 130 miles.

... the earth moves 69000 miles in its orbit in an ...h, what space does it move at the same rate in ...econds? Ans. 19320 miles.

... Jamaica is said to produce 70000 tons,

10*cwt.* 2*qrs.* of sugar: what is their value at $7·25 per hundred weight? Ans. $10,150,076·12½

21. What quantity of shalloon that is 3*qrs.* of a yard wide will line 7*yds.* 2*qrs.* of cloth that is 1*yd.* 2*qrs.* wide?

Ans. 15*yds.*

22. How many yards of matting, 2 feet 6 inches broad, will cover a floor 27 feet long and 20 feet broad?

Ans. 72*yds.*

133. If there be a remainder after the division by the first term, it is of the same denomination as the third term; and, if it admit, it may be reduced lower, and the operation continued as in Compound Division.

Exam. 4. If a piece of linen containing 26*yds.* cost £3, what would 732*yds.* cost at the same rate?

```
    yds.  yds.   £
As   26 : 732 :: 3
              3
           ----   £   s.  d.
     26)2196(84   9   2½
        208
        ----
         116
         104
         ---
          12
          20
         ---
         240 shillings.
         234
         ---
           6
          12
          --
          72 pence.
          52
          --
          20
           4
          --
          80 farthings.
          78
          --
           2
```

Here, after we have found £84, the first part of the answer, there is a remainder of £12, which being reduced to shillings, the rest of the operation proceeds as in Compound Division, and there is a final remainder of 2 farthings.

135. When the first term is an unit, the answer is found by multiplying the second and third: and when the second or third term is unity or 1, the answer or fourth term is found by dividing the other term by the first.

Ex. 5. How many barrels of flour can I purchase for \$748, at the rate of \$5·50 per barrel?

```
      cts.    $     bar.
As    550 : 748 : : 1 :
            100
          -----
    550)74800(136 barrels, answer.
        550
        ----
        1980
        1650
        ----
         3300
         3300
         ----
```

Here, the first and second terms being of the same denomination; that is, 550 cents, and 74800 cents, the answer is evidently found by dividing the second by the first, because the third term is unity.

23. If the rent of 5 acres be £4 13*s.* 4*d.* how much land could be rented at the same rate for £70 10*s.* 6*d.**

Ans. 75 acres, 2 roods, 10 poles.

24. How many yards of cloth can I buy for \$1876·50, at \$5·25 per yard? Ans. 357*yds.* 1*qr.* $2\frac{6}{7}$*n.*

25. If 39*cwt.* 3*qrs.* 20*lbs.* of pork cost \$160, what would 1*cwt.* cost? Ans. \$$4\frac{4}{559}$, or \$4 nearly.

26. If 1*cwt.* of beef cost \$5·25, what will 39*cwt.* cost at the same rate? Ans. \$204·75.

27. If 1*lb.* of butter cost $18\frac{3}{4}$ cents, what will 2*cwt.* 2*qrs.* 12*lbs.* cost? Ans. \$54.75.

28. Bought a cask of sugar weighing 7*cwt.* 3*qrs.* 25*lbs.* at \$8·75 per *cwt.* What is the amount? Ans. \$$69·76\frac{63}{112}$.

* In this exercise, the first and second terms are first reduced to the same denomination, and then the work proceeds as in the last example.

29. What will the carriage of 17*cwt.* 3*qr.* 12*lb.* come to at 87½ cents per *cwt?* Ans. 15·62½

30. If 1 gallon of wine cost $1·75, what will 1 tun cost at the same rate? Ans. $441.

135. When the third term is of more denominations than one, it is generally necessary, or at least proper, to reduce it to the lowest denomination mentioned in it.

When the first and second terms are not very large, however, it is often better not to perform this reduction, but to employ compound multiplication and division. And, in fact, when one of the terms is unity or 1, it is in general better to employ compound multiplication or division as the case may require.

Example 6. If a barrel of gin containing 31 gallons, 2 pints, cost $9·25, how many gallons can I purchase for $925 at the same rate?

Here the third is reduced to pints, the lowest denomination mentioned in it. Then by the usual process, the answer is found to be 25000 pints, and this, by proper reduction, becomes 3150 gallons.

```
    cts.     cts.      gal. pt.
As 925 : 92500 : :  31   2
           250         8
       ---------     ----
         4625000      250
        185000
       ---------      pts.
 925) 23125000(25000
        1850
        ----
         4625
         4625
         ----
          000
         ----
```

Exercises.—31. If 63 gallons of wine cost $118·12½, what cost 10 gallons? Ans. $18·75.

32. If 1 pipe of Holland gin, containing 120 gallons, 8 quarts, cost $118; what quantity can I buy for $1·37½? Ans. 1*gal.* 1[illegible]*qt.*

33. If a man walk 7 miles, 2 furlongs, 30 poles, in 2 hours, 10 minutes, how many miles will he walk at the same rate in 13 hours? Ans. 44*m.* 0*fur.* 2*p.*

34. The mean daily motion of the Earth in its orbit is

3″; in what time will it perform its tropical revolution d the sun; suppose the orbit to be divided into 360°?
Ans. 364 *dys.* 18*hrs.* 17*min.* $31\frac{2597}{3553}$sec.

136. The work is often much abbreviated by dividing the first and second, or the first and third terms, out never the second and third,) by any number which exactly measures them, and employing the quotients instead of the numbers themselves.

Example 7. If fifteen yards of linen cost $17·95, what will 55 yards cost at the same rate?

Here, the first and second terms being divided by 5, which a common measure, we obtain 3 and 11, which are to be used instead of 15 and 55.*

	yds.	*yds.*	*cts.*
As	15 :	55 : :	1795 :
			11
	3 :	11	3)19745
		Ans.	$ 65.81¾

Exercises.—35. If a lot of sugar containing 56*cwt.* cost £214 18*s.* 4*d.* what does it cost per ton? Ans. £76 15*s.* 1¼*d.* rem. $\frac{5}{7}$*far.*

36. If a servant's wages be 125 dollars a year, how much has he to receive for 73 days' service? Ans. $25.

37. If a servant receives $87·50 for 30 weeks' service; how long ought he to remain in his place for $204·16⅔? Ans. 70 weeks.

38. If a carpenter can earn $9·75 in 6 days; how much can he earn in 300 days? Ans. $487·50.

39. Admitting that a man pays for board, lodging, &c. $22·75 a month, of 30 days: what does he lay out in 365 days at the same rate? Ans. $276·79⅙.

40. What cost 79*cwt.* of pork, if 7*cwt.* cost $31·50? Ans. $355·50.

41. What cost 7 tons, 10*cwt.* of pot ashes, first sort, at … per ton? Ans. 656·25.

…at cost 10 tons, 5*cwt.* 3*qrs.* of pot ashes, pearl, at …n? Ans. $951· 59⅜.

…is contraction is plain from what has been laid …o of two numbers is the same as the ratio of …ed by multiplying or dividing them by the … to the same thing, if the divisor and … the same number, the quotient is.

43. If 7 tons of pot ashes, Barilla, cost $262·50, what will 9*cwt.* 3*qrs.* come to at the same rate? Ans. $18·28.1$\frac{1}{4}$.

44. What cost 76 pounds of beeswax, at 28$\frac{1}{2}$ cents per pound? Ans. $21·66.

45. If 6 gross of Bristol porter bottles cost $57, what will 21 gross cost? Ans. $199·50.

46. What cost 215 pounds of bread, navy, at 90 cents for 14 pounds? Ans. $13·82·1$\frac{3}{7}$.

47. What cost 196 pounds of spermaceti candles at 31$\frac{3}{4}$ cents per pound? Ans $62·23.

48. If 7 pounds of clover seed cost 87$\frac{1}{2}$ cents, what will 112 pounds cost? Ans. $14·00.

49. What will 700 chaldrons of Liverpool coal cost, at $43·56$\frac{1}{4}$ for 4 chaldrons and 9 bushels? Ans. $7175.

50. What will 35 chaldrons of Schuylkill coal cost, at $108 for 9 chaldrons? Ans. $420·00.

51. If a bag of coffee, Java, weighing 110 pounds, cost $18·15, what will 14 pounds cost? Ans. 2·31.

52. What cost 7*cwt*, 3*qrs.* 19*lbs.* of American cordage at $12·50 per *cwt.*? Ans. $98·99$\frac{62}{112}$.

53. If 1 pound of cotton, New-Orleans, cost 11$\frac{3}{4}$ cents, what is 1976 pounds worth at the same rate?

Ans. $232·18.

54. Bought 12 yards of blue calico for $2·10; how much will 500 yards cost, at the same rate? Ans. $87·50.

55. If 1*cwt.* of brimstone, roll, cost $2·37$\frac{1}{2}$, what will 13 pounds cost? Ans. $0.27·5$\frac{75}{112}$.

56. Bought 3 pipes of brandy, containing 120, 121$\frac{1}{2}$, and 124$\frac{3}{4}$ gallons, respectively, at $1·18$\frac{3}{4}$ per gallon; what is the cost? Ans. $434·92$\frac{3}{16}$.

57. An honest tradesman, through unforseen misfortunes, is obliged to call his creditors together; he finds his debts to be £4326, and he can pay 14*s.* 6*d.* in the pound; how much has he still left? Ans. £3136 7*s.*

58. How much will 750 pounds of feathers, American, cost at 31$\frac{3}{4}$ cents per pound? Ans. 238·12$\frac{1}{2}$.

59. How much will 87 pounds of feathers, foreign, cost at 21$\frac{1}{2}$ cents per pound? Ans. $18·70$\frac{1}{2}$.

60. Bought 5 barrels of Baltimore flour for $28·75; how many barrels can I buy for $2875? Ans. 500*b.*

61. If 3 casks of raisins, Malaga, cost $21·75, how many casks can I buy for $58? Ans. 8 casks.

62. If 6 bushels of wheat, Genesee, cost $6·75, how many bushels can I buy for $1000 ?

Ans. 888*bush.* 3*pecks*, $4\frac{12}{27}$*qts.*

63. If 2 barrels of beef, mess, cost $18·50, what will 101 barrels cost ? Ans. $934·25.

64. How much will 360 bushels, 2 pecks of salt, Liverpool ground, cost at $46\frac{1}{2}$ cents per bushel ?

Ans. $$167·63\frac{1}{4}$.

65. If 1 gallon of Jamaica rum, 4th proof, cost $$1·12\frac{1}{2}$, what will 126 gallons, 2 quarts cost ? Ans. $$142·31\frac{1}{4}$.

66. What will 7*cwt.* 3*qrs.* 11*lbs.* of Havana sugar, white, cost at $12\frac{3}{4}$ cents per pound ? Ans. $$112·07\frac{1}{4}$.

67. If 13 pounds of loaf sugar cost $$2·27\frac{1}{2}$, what will 112 pounds cost ? Ans. $19·60.

68. If a clerk have a salary of $400 per year, commencing at the first of May, how much has he to receive on leaving his situation on the 18th of December, both days included ? Ans. $254·24·6\frac{42}{73}$.

69. If the digging of a mile of canal cost $6500, what will the digging of 30 miles, 7 furlongs, and 21 poles cost ?

Ans. $$201114·06·2\frac{1}{2}$.

70. If a person travelling 14 hours per day, finish the first half of a journey in 9 days, in what time will he finish the remaining half, travelling 10 hours per day at the same rate ?

Ans. $12\frac{3}{5}$*dys.*

71. How many American miles are equivalent to 128 Irish; 11 Irish miles being equivalent to 14 American ?

Ans. 162*m.* 7*fur.* 10*p.* 5*yds.*

72. The mean length of a degree on the Earth's surface is 69 American miles and 80 yards, nearly ; how much is it in Irish miles ? Ans. 54*m.* 2*fur.*

73. It is found, that the diameter of every circle is to its circumference very nearly in the ratio of 113 to 355 ; what is the diameter of a round tree, if the girt be 6 feet 10 inches ?

Ans. 2 feet, $2\frac{36}{355}$*in.*

74. The Earth revolves round the sun in an orbit, the mean diameter of which is 190000000 of American miles. Were this orbit an exact circle, as it nearly is, what would be its circumference ? Ans. $596902654\frac{88}{113}$ miles.

75. The length of a wall, being tried by a measuring line, appears to be 1287 feet, 4 inches ; but on examination, the line is found to be 50 feet, $10\frac{1}{2}$ inches in length, instead of

50 feet, its supposed length. Required the true length of the wall. Ans. 1265*ft.* 2$\frac{188}{407}$*in.*

76. If a shop-keeper use a false weight of 14$\frac{3}{4}$ ounces for a pound, how many pounds will 112 pounds of just weight appear to be, when weighed by his weight?

Ans. 121$\frac{29}{59}$*lbs.*

77. If the port wine contained in a vessel, weigh 15*cwt.* 3*qrs.* 24*lbs.* how much weight of pure water, and how much of cow's milk, would the same vessel contain, the weight of equal bulk of port wine, of water, and of milk, being as the numbers 997, 1000, and 1032?

Ans. 16*cwt.* 0*qr.* 1$\frac{379}{997}$*lbs.* and 16*cwt.* 2*qr.* 2$\frac{766}{997}$*lbs.*

78. If a stone column weigh 69*cwt.* 3*qr.* 10*lbs.* what will a pillar of oak of the same dimensions weigh, the weight of equal bulks of the stone and oak being as the numbers 2484 and 1170?

Ans. 32*cwt.* 3*qrs.* 16$\frac{19}{69}$*lbs.*

79. In what time would wind move from the pole to the equator, at the rate of 2$\frac{3}{4}$ miles per hour, the distance being 6214 miles? Ans. 94*dys.* 3*hrs.* 38*min.* 10$\frac{10}{11}$*sec.*

80. The Earth describes its orbit round the sun in 365 days, 5 hours, 48 minutes, 48 seconds: through what space in American miles does it move each hour, at an average, the circumference of the orbit being 596902655 miles, nearly? Ans. 68094$\frac{756423}{1972308}$.

81. The ecliptic contains 360 degrees, and is apparently described by the sun in 365 days, 5 hours, 48 minutes, and 48 seconds, through what part of it does he appear to move at an average each day, and each hour?

Ans. $59'\ 8''\ 19'''$, and $2'\ 27''\ 50'''$, nearly.

82. In 1798, the quantity of sugar imported into England from the West Indies, was 2361715*cwt.* 0*qr.* 8*lbs.*; the duty paid on which was £2070377 2*s.* 7*d.* What was the duty per *cwt.*? Ans. 17*s.* 6$\frac{13024844}{33064011}$*d.*

83. How many yards 3*qrs.* wide are equal in measure to 30 yards, 5*qrs.* wide? Ans. 50*yds.*

84. If 4*cwt.* 2*qrs.* of merchandise be carried 36 miles for $3, how many pounds can I have carried 20 miles for the same money? Ans. 907*lbs.* 3*oz.* 3$\frac{1}{5}$ *drams.*

85. How much in length, that is 12$\frac{1}{2}$ poles in breadth, must be taken to contain an acre?

Ans. 12 *poles,* 4*yds.* 1*ft.* $\frac{2}{5}$*in.*

86. A wall that is to be built to the height of 27 feet, was raised 9 feet by 12 men in 6 days; how many men must be

...ed to finish the wall in 4 days, at the same rate of ...g? Ans. 36 men.

If a tailor can make a coat and waistcoat with 3 yards ... quarters of cloth which is 1 yard, 2 quarters in ...dth; how many yards will he require to make the ..., when the breadth is only 2 quarters, 2 nails? Ans. 9*yds.*

88. If there are in a garrison provisions for 1500 men ... weeks, which, on account of the rains, is seven weeks ...nger than the siege can last: how many soldiers may be ...rought in to defend the place for 3 weeks, without lessening the quantity of food to any individual? Ans. 3500 men.

89. If a garrison, containing 22400 men, have provisions to last 3 weeks, how many inhabitants must be sent away, so as to make the same provisions last 7 weeks? Ans. 12800 men.

90. If 12 inches in length and 12 in breadth make a square foot: what length of board, 8 inches broad will be equal to the same measure? Ans. 18*in.*

91. If 220 yards in length, and 22 in breadth, make an acre: what must be the breadth when the length is 121 yards? Ans. 40*yds.*

137. It may be remarked that, in weighing merchandise, the weight of any commodity together with the weight of the box, barrel, &c. which contains it, is called its *gross weight.*

The weight of the box, barrel, &c. usually called the *tare*, must be deducted from the gross weight in order to find the *neat weight.*

The *tare* is also sometimes allowed either at so much ...ndredweight, and so much in the gross weight. ...e places there is an allowance of 4 pounds in ...ounds on merchandise liable to waste, of ...fter the tare is deducted, which is called

...d; what remains after the tare is deducted the *suttle weight*; and the *neat* or ... deductions have been made. ...ces vary; in some places there is ... 3 hundredweight of what re... This allowance is for the following examples and ...he mode in which de-

Ex. 8. Required the neat weight of 39*cwt.* 2*qrs.* 20*lbs.* tare 14*lbs.* per *cwt.*

Here, the numbers being arranged as in the margin, the operation is much abbreviated by dividing 112 and 14, by 14, which is a common measure; (*see* §112,) the quotients are 8 and 1; which are to be used instead of 112 and 14; and the operation is still abbreviated by employing compound division. The neat weight is found to be 34*cwt.* 3*qrs.* 21*lbs.*

	lbs.		*lbs.*		*cwt.*	*qrs.*	*lbs.*
As	112	:	14	: :	39	3	20
	——		—		4	3	27
	8		1				
					34	3	21

Ex. 9. Find the neat weight of 39 barrels of sugar, weighing gross 83*cwt.* 1*qr.* 10*lbs.*, tare 18*lbs.* per barrel, tret 4*lbs.* in every 104*lbs.*

Here, by multiplying 39 by 18, the tare is found to be 702*lbs.*, which, by reduction, is equivalent to 6*cwt.* 1*qr.* 2*lbs.* This subtracted from the gross weight, leaves a remainder of 77*cwt.* 0*qrs.* 8*lbs.* Then, by proceeding as in the margin, the neat weight is found.

					cwt.	*qrs.*	*lbs.*
				From	83	1	10
				Take	6	1	2
	lbs.		*lbs.*				
As	104	:	4	: :	77	0	8
	——		–		2	3	24
	26		1				
					74	0	12

Example 10. The neat weight of 4 hogsheads of sugar, tret as usual, the gross weight and tare as follows:

			cwt.	*qr.*	*lb.*		*qr.*	*lb.*
		No. 1.	9	3	20	tare	2	15
		2.	10	1	14		2	20
		3.	10	2	18		2	25
		4.	10	3	8		3	8
Gross weight			41	3	4	tare 2	3	12
		tare	2	3	12			
lbs.	*lbs.*							
As 104 :	4 : :		38	3	20			
——	–		1	1	$27\frac{9}{13}$			
26	1							
		Neat	37	1	$20\frac{4}{13}$,			

Here, the gross weight of the four hogsheads is found, by compound addition, to be 41 hundredweight, 3 quarters,

4 pounds; and the tare, 1 hundred weight, 3 quarters, 12 pounds. Subtracting the tare from the gross weight, the result is the suttleweight; then by arranging the quantities, as in the last example, and proceeding by compound division, we find the neat weight to be 38 hundredweight, 1 quarter, 16 pounds.

Exercises.—92. In 25 barrels of figs, each 2*cwt.* 1*qr.* gross, tare 16*lbs.* per *cwt.*, how much neat?

Ans. 48*cwt.* 0*qr.* 24*lbs.*

93. In 152*cwt.* 1*qr.* 3*lbs.* gross, tare 10*lbs.* per *cwt.*, and tret as usual; how much neat?

Ans. 133*cwt.* 1*qr.* 10*lbs.* 13$\frac{38}{91}$*oz.*

94. What is the value of 13*hhds.* of tobacco, at $7·50 per *cwt.* each weighing 4*cwt.* 3*qrs.* 17*lbs.* gross, tare 13*lbs.* per *cwt.*?

Ans. $422·45·$\frac{485}{6272}$.

95. Find the neat weight of 22 bags, weighing gross 45*cwt.* 1*qr.* 19*lbs.*, tare 5*lbs.* per bag, tret as usual.

Ans. 42*cwt.* 2*qrs.* 25$\frac{15}{26}$*lbs.*

96. Find the neat weight of 35 barrels of anchovies, each weighing 1*qr.* 12*lbs.*, tare 14*lbs.* per *cwt.*

Ans. 10*cwt.* 3*qr.* 21*lb.*

97. What is the value of 26*hhds.* of tobacco, at $7·75 per *cwt.* neat weight, each weighing 4*cwt.* 2*qrs.*, tare 13*lbs.* per *cwt.*?

Ans. $801·50·2$\frac{13}{56}$.

98. What is the price of 50 casks of butter, gross weight 45*cwt.* 1*qr.* 25*lbs.*, tare 15*lbs.* per cask, at $21 per *cwt.* neat weight?

Ans. $814·50.

99. I want to know the height of a tree, by means of the length of its shadow; I set up a straight stick that measures above the ground 3 feet, 4 inches; the shadow of this is 5 feet, 2 inches, and the shadow of the tree, at the same moment, I find to be 79 feet, 10 inches. Ans. 123*ft.* 8$\frac{9}{10}$*in.*

Questions.

What is simple proportion?

What is its object?

What is compound proportion?

Repeat the rule for finding a fourth proportional to three given numbers.

What is direct proportion?

What is inverse proportion?

there be a remainder after dividing the product of the third by the first term, how do you proceed?

When the first term is an unit, how is the answer found?

When the second or third term is an unit, how is the answer found?

When the first and second terms are not of the same denomination, how are you to proceed with the operation?

What is to be done with the third term, when it is of more denominations than one?

When the first and second, or the first and third terms, admit of a common measure, how is the process to be performed?

Compound Proportion.

138. Rule I.—By the rule for simple proportion, find a fourth proportional to two given terms of the same kind with one another, and to the term which is of the same kind as the answer. To two other terms of the same kind, and to the last obtained, find a fourth proportional; and thus proceed if there be more terms: the final result will be the answer.

139. Rule II.—Place the term which is of the same kind as the required term, in the last place. Comparing the other given terms by pairs, place each as antecedent or consequent, according to the rule for simple proportion. Divide the continual product of all the consequents, and the last term, by the continual product of all the antecedents; the quotient will be the answer.

As a contraction in the use of the latter rule, divide an antecedent and any consequent by any number that will measure them, and employ the results instead of those terms: or if an antecedent and any consequent, or an antecedent and the last term be the same, reject them.

Example 1. If 40 gallons of ale serve 17 persons for 5 days, how many gallons will serve 9 persons for a year, at the same rate?

I. As 5 days : 365 days : : 40 gal. : 2920 gal.
As 17 persons : 9 persons : : 2920 gal. : $1545\frac{15}{17}$ gal.

In this operation, we first proceed as if the number of persons in both cases were 17, and on this supposition we find that 2920 gallons would be used by those persons in a

year. But the number of persons being 9, instead of 17, we find by the second analogy, that if 17 persons would use 2920 gallons in a year, 9 persons would use $1545\frac{15}{17}$ gallons in the same time.

$$\text{As}\ \left.\begin{array}{lcl} 5\ \text{days} & : & 365\ \text{days} \\ 17\ \text{persons} & : & 9\ \text{persons} \end{array}\right\} :: 40\textit{gals.} : 1545\tfrac{15}{17}\textit{gals.}$$

In the second method that term is placed last which is of the same kind as the required term. Thus, were the number of persons the same, it is evident that the answer would be greater than 40 gallons; and therefore we put 365 days in the second and 5 days in the first place; but were the number of days the same, it is obvious that the required term would be less than 40 gallons; and therefore we put 9 persons in the second place and 17 in the first. We then multiply the product of the consequents by the common third term, 40, and divide the result by the product of the antecedents, and the same answer is found as before. This operation is in effect the same as that in the first method; for the result of the first single analogy, without the actual multiplication and division, is $\frac{365\times40}{5}$, and, consequently, the second analogy becomes $17 : 9 :: \frac{365\times40}{5} : \frac{9\times365\times40}{5\times17}$—whence it appears that in both methods the same multiplications and divisions are in reality performed, and consequently the one is only a modification of the other. In this method, 5 and 365, or 5 and 40, might be divided by 5 as a contraction.*

Exam. 2. If a family of 13 persons spend \$64 on butcher's meat in 8 months when the meat is 6 cents per pound, how much, at the same rate, should a family of 12 persons spend in 9 months, when the meat is $6\frac{1}{2}$ cents per pound?

$$\text{As}\ \left.\begin{array}{lcl} 13\ \text{persons} & : & 12\ \text{persons} \\ 8\ \text{months} & : & 9\ \text{months} \\ 6\ \text{cents} & : & 6\tfrac{1}{2}\ \text{cents} \end{array}\right\} :: \$64 : \$72$$

Here the last ratio being the same with that of 12 : 13,

* The principles on which the operations in Compound and in Simple Proportion depend are the same.

It may be remarked that one very considerable advantage which the second rule possesses, is, that by it the operation is kept entirely free from fractions till the conclusion; while in the other mode fractions often arise from the first analogy, and render the remaining part of the work more intricate and difficult.

the terms of the first and last ratios may be erased, and therefore as 8 : 9 :: \$64 to the answer, which is known by inspection to be \$72.

Exam. 3. If the carriage of 66*cwt.* for 42 American miles be \$30, what is the carriage of 36 hundreds, *long weight*, (see § 37, 40,) for 90 Irish miles, at the same rate?

$$\left.\begin{array}{lcl} \text{As } 66\,cwt. & : & 36 \text{ long cwt.} \\ 112\,lb. & : & 120\,lb. \\ 42\,m. & : & 90\,m. \text{ Irish.} \\ 11 & : & 14 \end{array}\right\} :: \$30 : \$46{\cdot}63\tfrac{439}{847}$$

In this example, since the answer is to be in money, \$30 is placed as the common third term. Then it is evident that were all things alike except the number of hundreds, the answer would be less than \$30; and therefore 36 is put as consequent and 66 as antecedent. But had the former number of hundreds been equal, the answer must have been greater than \$30, on account of the difference in their magnitudes; and therefore 112 is made antecedent and 120 consequent. The arrangement of the remaining terms proceeds on similar principles, and the answer is found by dividing the continual product of \$30 and the consequents by the continual product of the antecedents.

The operation is much abbreviated, as in the margin, by dividing the terms of the first and third ratios by 6, and those of the second by 8, and neglecting in the work 14, which occurs as an antecedent in one ratio, and a consequent in another.*

$$\left.\begin{array}{rcl} 11 & : & 6 \\ (14) & : & 15 \\ 7 & : & 15 \\ 11 & : & (14) \end{array}\right\} :: \$30 : \$46{\cdot}63\tfrac{439}{847}.$$

Exercises.

1. If 3 masons, working 7 hours a day, build a wall in 6 days, how many hours a day must 4 masons work in order to build it in 5 days? Ans. 6 hours, 18 minutes.

* This question might also have been wrought by four analogies in Simple Proportion. Every one of these, however, would have given origin to fractions, the neglecting of which would have prevented the precise result from being obtained. The work might also have been modified by reducing the hundreds and the miles, respectively, to the same kind. The using 112, indeed, as an antecedent and 120 as a consequent, serves this purpose, as the hundreds are thus reduced to pounds: and the multiplication by 11 and 14 serves a similar purpose in relation to the miles.

2. If 9 bushels of corn serve 7 horses 10 days, how many bushels at the same rate will serve 20 horses 21 days? Ans. 54 bushels.

3. If a family of 19 persons expend \$235 in 8 months, how much at the same rate will a family of 12 persons expend in 5 months? Ans. \$92·76·3$\frac{3}{19}$.

4. If 96 men, working 9 hours for 10 days, can dig a trench 400 yards long, 3 wide, and 2 deep, in how many days at the same rate can 108 men, working 7 hours a day, dig a trench of 175 yards long, 4 wide, and 3 deep?
Ans. 10 days.

5. If the carriage of 59*cwt.* 19 miles cost £2 16*s.* how how far may 43*cwt.* be carried at the same rate for £2 4*s.*?
Ans. 20$\frac{291}{602}$ miles.

6. If the carriage of 13*cwt.* 65 miles cost \$9, how many hundreds may be carried 40 miles at the same rate for \$15?
Ans. 35$\frac{5}{24}$*cwt.*

7. If 12 oxen in 5 days plough 11 acres, how many oxen would plough 38 acres in 18 days? Ans. 10.

8. If a person walking 12 hours each day, perform a journey of 250 miles in 9 days, in how many days, walking 10 hours each day at the same rate, would he complete a journey of 400 miles? Ans. 17$\frac{7}{25}$ days.

9. If the expenses of a family of 8 persons amount to \$42 in 16 weeks, how long will \$100 support a family of 6 persons at the same rate? Ans. 50$\frac{50}{63}$ weeks.

10 If 29 men, in 5 days of 12 hours each, reap 32 American acres of wheat, in how many days of 13 hours each, will 20 men, working equally, reap 40 Irish acres?
Ans. 13$\frac{866}{1573}$ days.

11. If \$312 pay 16 labourers for 18 days, how many labourers, at the same rate, will \$702 pay for 24 days?
Ans. 27.

12. If 36 yards of cloth, 7 quarters wide, cost \$504, what cost 120 yards of the same quality, but only 5 quarters wide? Ans. \$1200.

13. If a tradesman earn 16 guineas in 108 days, how many sovereigns would he earn, at the same rate, in 270 days; 20 guineas being equivalent to 21 sovereigns?
Ans. 42.

15. If 3000 copies of a history of the United States, each containing 11 sheets, require 66 reams of paper, how much paper will 5000 take if the work be extended to 12$\frac{1}{2}$ sheets? Ans. 125 reams.

15. If a puncheon of rum, containing 85 gallons, cost £58 8s. 9d. what would be the value of a hogshead, containing 63 gallons, and composed of four parts of the same rum, and one part of water? Ans. £34 13s.

16. If a person walking 13 hours each day, travel 191 in 7 days; in how many days of 10 hours, will he complete the remainder of a journey of 500 miles, at the same rate each hour? Ans. $14\frac{1379}{1910}$ days.

17. If 63 pounds of tea cost $52, what cost 70 pounds of a different quality, 9 pounds of the former being equal in value to 10 pounds of the latter? Ans. $52.

18. If in 4 months I spend as much as I gain in 3 months, how much do I lay up at the years' end, if I gain every 6 months $428·50? Ans. 214·25.

19. If 100 dollars in 12 months gain 6 dollars interest, how much will $75 gain in 9 months? Ans. $3·37·5.

20. Suppose 30 men perform a piece of work in 20 days; how many will accomplish another piece of work 4 times as large in $\frac{1}{5}$ of the time? Ans. 600 men.

Questions.

In arranging the terms of a question in compound proportion, according to Rule I., which of the terms must be placed in the third place?

Which terms must be placed in the first and second places?

After a fourth proportional is found to three of the given numbers, how do you proceed?

How are numbers to be arranged according to Rule II.?

Having the given quantities properly arranged, how is the operation to be performed?

CHAPTER VIII.

Interest, Discount, Commission, Insurance, &c.

140. The sum to be paid by a person for the use of money which he owes, is called the *interest* of that money.

The money due is called the *principal*.

The sum of the principal and interest is called the *amount*.

The *rate* is the money allowed for the use of one hundred dollars for any given time, but usually for a year.

When interest is charged on the *original principal only*, it is termed *Simple Interest*.

When interest is charged, not only on the original principal, but also *on the interest as it becomes due*, it is called *Compound Interest*.

*Simple Interest.**

PROBLEM I.—*To find the interest of a given sum, at a given rate per cent. per annum.*†

141. RULE.—As the principal 100 dollars or pounds is to the rate per cent. per annum, so is the given principal to its interest for one year; or, which amounts to the same thing, multiply the principal by the rate, and divide the product by 100.

Example 1. Required the interest of $756 for 1 year, at 6 per cent. per annum.

$	$	$
As 100 :	6 ::	756 :
		6
		100)4536
		$ 45·36

The reason of the operation is quite evident, as it is nothing more than this: as the principal, $100, is to its interest, so is the principal, $756, to its interest; and it is evident, that as often as the one principal contains its interest, so often will the other contain its interest: that is, by the nature of proportion, the interest will be proportional to the principal.

* In interest five quantities are concerned, the *principal*, the *rate*, the *time*, the *interest*, and the *amount*, and any three of these, except the principal, the interest, and the amount, being given, the rest can be found. Hence, calculations in interest admit of several problems. The most useful however, and consequently that which claims the greatest degree of attention, is that in which the principal, the time, and the rate, are given to find the interest or amount.

† It is scarcely necessary to remark, that *per cent.* means *per hundred*, and *per annum*, *per year*. The legal rate of interest in the United States is at present 7 per cent.

Required the interest of £576 5*s*. 8½*d*. for 1 year, at 6 per cent. per annum.*

Here, the work is much abbreviated, by not reducing the third term to the lowest denomination (halfpence) mentioned in it, but by proceeding as in the margin, in multiplying the third term by the second, and then dividing the result by 100: we thus find the interest to be £34 11*s*. 6¼*d*. and 4 farthings of a remainder,

	£		£		£	*s*.	*d*.
As	100	:	6	::	576	5	8½
							6
				100)	34|57	14	3
					20		
					11|54		
					12		
					6|51		
					4		
					2|04		

Exam. 3. Required the interest and amount of $325·75 for 1 year, at 7 per cent. per annum.

Here, the principal is $325·75, and the interest is found to be 2280¼ cents, or $22·80¼: hence the amount, or the sum of both is $348·55¼.

	$		$		*cts*.
As	100	:	7	::	32575
					7
				100)	228025
					2280¼ cents
					$22·80¼

Exercises.

1. Required the interest of $87·87½ for 1 year, at 6 per cent. per annum. Ans. $5·27¼.

2. Required the interest of $7000 for 1 year at 7 per cent. per annum. Ans. $490.

* The rate of interest has varied much at different periods, and in different countries, but it has been generally observed to diminish as commerce extends. In Italy, about the beginning of the 13th century, it varied between 20 and 30 per cent; and in the Netherlands it was fixed by Charles V. in 1560 at 12 per cent. By an act of the 37th year of Henry VIII. interest in England was not to exceed 10 per cent. By 21st James I. it was reduced to 8 per cent. Soon after the Restoration it was reduced still farther to 6 per cent.; and in the 12th of Anne to 5 per cent. the present rate. The legal rate of interest in Ireland is at present 6 per cent. per annum.

3. Required the interest of \$3500 for 1 year, at $5\frac{1}{2}$ per cent. per annum. Ans. \$192·50.

4. Required the interest of \$137·75 for 1 year, at $7\frac{3}{4}$ per cent. per annum. Ans. \$10·67$\frac{1}{2}$.

5. Required the amount of \$2500 for 1 year, at 7 per cent. per annum. Ans. \$2675.

6. Required the amount of \$875 75 for 1 year, at $6\frac{1}{2}$ per cent. per annum. Ans. \$931·87$\frac{1}{2}$.

PROBLEM II.—*To find the interest of a given principal for any other time than a year.*

142. RULE I.—Find the interest for a year by Rule I. then, as 1 year is to the given time, so is the interest for 1 year to the interest required.*

Example 5. Required the interest of 550 dollars for $3\frac{1}{2}$ years at 7 per cent. per annum.

Here, in the first place, the interest for 1 year is found by the Rule I. to be \$38.50: and the interest for $3\frac{1}{2}$ years, is readily found from the second analogy, to be \$134.75.

	\$		\$		\$		\$
As	100	:	7	::	550	:	38.50
	Year.		*Yrs.*		\$		\$
As	1	:	$3\frac{1}{2}$	::	38.50	:	134.75

143. RULE II.—The given rate per cent. per annum must be to the interest sought in a ratio compounded of the ratios of the principals and times: hence, this problem can be easily solved by Rule II. Compound Proportion.

Example 2. At 5 per cent. per annum what is the interest of £275 10*s.* for $3\frac{1}{2}$ years?

	£		£		£		£ *s.* *d.*
As	100	:	$275\frac{1}{2}$	} ::	5	:	48 4 3
	1 year	:	$3\frac{1}{2}$ years	}			

It is plain, that there is given the interest of £100 for 1 year, in order to find the interest of £275 10*s.* at the same rate for $3\frac{1}{2}$ years. The third term of the analogy, therefore, must be the given interest £5; and this must be the

* The work may often be abbreviated by finding the interest for months or fractional parts of a year, by the method of aliquot parts. This shall be illustrated in a subsequent chapter, which will treat of abridged methods of calculating the price of commodities; also, interest, insurance, commission, &c. In using this method, the answer will often be found with more ease, or with a greater degree of correctness by multiplying by the rate; then multiplying or taking aliquot parts the time; and last of all dividing by 100.

interest sought in a ratio compounded of 100 : $275\frac{1}{2}$ and of 1 : $3\frac{1}{2}$, or of 200 : 551 and of 2 : 7; that is, in the ratio of 400 : 3857. The answer therefore is $\frac{3857 \times 5}{400}$, or $\frac{3857}{80}$, that is £48 4*s.* 3*d.**

Exam. 3. At 7 per cent. per annum, what is the interest of \$500 for $2\frac{1}{2}$ years?

$$\left.\begin{array}{l}\text{As } \$100 : \$500 \\ 1 \text{ year} : 2\tfrac{1}{2} \text{ years}\end{array}\right\} :: \overset{\$}{7} : \overset{\$}{87.50}$$

Here, the third term of the analogy, \$7, is to the interest sought in a ratio compounded of 100 : 500 and of 1 : $2\frac{1}{2}$, or of 1 : 5 and 2 : 5, that is, in the ratio of 2 ; 25. The answer, therefore, is $\frac{25 \times 7}{2} = \frac{175}{2} =$ \$87.50.

Exercises.—Find the interests of the following principals for the given times, and at the given rates per cent. per annum.

Exercises.	*Answers.*
1. 750 dollars for 2 years 5 months, at $4\frac{3}{4}$	\$86.09.3+.
2. 275 dollars for $8\frac{1}{2}$ months, at $5\frac{1}{4}$	10.22.6+
3. 72.35 for 1 year 8 months, at 7	8.44.+
4. 2005.75 for 6 years, at 6	722.07.
5. 365 for 63 days, at 6	3.78.
6. 100.25 for 63 days, at 7	1.21.1+
7. 3756.75 for 63 days, at 5	32.42.1+
8. £812 10*s.* 10*d.* for 2 years 5 months, at $4\frac{3}{4}$	£93 5 $4\frac{3}{4}$
9. £250 18*s.* 4*d.* for 1 year 9 months, at 5	21 19 $1\frac{1}{4}$
10. £651 0*s.* 0*d.* for 7 months, at $4\frac{1}{2}$	17 1 $9\frac{1}{4}$

11. Required the amount of 1756 dollars 75 cents, from June 29, 1824, till February 12, 1827, at 7 per cent. per annum. Ans. \$2079.51.$0\frac{1}{2}$

12. Required the interest and amount of \$275.50, from August 1st, 1826, till April 19th, 1827, at 6 per cent. per annum. Ans. \$287.32 *amt.* \$11.82 *int.*

* In this manner, though often not the most expeditious, the learner ought for some time to calculate all questions in interest; and to prove his answer by such questions as the following: At what rate per cent. per annum, will the interest of 275*l.* 10*s.* for $3\frac{1}{2}$ years be 48*l.* 4*s.* 3*d.*? or, At 5 per cent. per annum, what principal will gain 48*l.* 4*s.* 3*d.* interest in $3\frac{1}{2}$ years? or, in what time will 275*l.* 10*s.* gain 48*l.* 4*s.* 3*d.* interest? And in some of those forms, persons who have been for years calculating interest by the common technical rules, are quite at a loss how to set about the solution; while children rationally taught for a very few months have found no difficulty in the question.

13. Required the amount of \$6000, from May 22d, 1826, 'till March 17th, 1827, at $4\frac{3}{4}$ per cent. per annum.

Ans. \$6233.46.5.

14. What is the interest and amount of \$374, from the 27th of May, 1819, till the 19th of April, 1827, at 7 per cent. per annum. Ans. 580.78.6 *amt.* 206.78.6 *int.*

PROBLEM III.—*To find what principal, in a given time, would produce a given interest, at a given rate per cent. per annum.*

144. RULE.—

$$\left.\begin{array}{r}\text{As rate} : \$100\\ \text{given time} : 1 \text{ year}\end{array}\right\} :: \text{interest} : \text{principal.}$$

Example.—How much money must be lent me on the 2d of April, at 7 per cent. per annum, to bring in for interest 35 dollars on the 18th of December following?

30 days in April,
2
——
28 April,
31 May,
30 June,
31 July,
31 August,
30 September,
31 October,
30 November,
18 December,
——
260 days from the 2d of April till the 18th of December.

$$\left.\begin{array}{r}\text{As } \$7 : \$100\\ 260 \text{ days} : 365 \text{ days}\end{array}\right\} :: \$35 : \$701.96.5.$$

Exercises.—1. What principal, at 5 per cent. per annum, will bring a yearly income of \$341.25? Ans. \$6825.

2. What principal lent on the 21st of May, 1827, till the 17th of November, 1828, at 6 per cent. per annum, will gain \$120? Ans. \$1337.04.7+

3. What principal, at 5 per cent. per annum, would be equivalent to the funded national debt of Great Britain and Ireland; the annual interest of the whole amounting to £27755546 9*s.* $1\frac{1}{2}$*d.* Ans. £555110929 2*s.* 6*d.*

4. What principal, at 7 per cent. per annum, would be equivalent to the national debt of the United States, suppose the annual interest amount to \$4935035? Ans. \$70500500.

PROBLEM IV.—*To find what principal, in a given time, would increase to a given amount, at a given rate per cent. per annum.*

145. RULE.—To the product of the time and rate, add the product of $100 and one year in the same name as the given time: Then, as the sum is to the above mentioned product of $100 and one year, so is the amount to the principal.

Exercises.—1. What principal lent on the 1st of January, 1827, at 6 per cent. per annum, would amount to $1000 on the 21st of May, in the same year? Ans. $980.39.2$\frac{8}{57}$.

2. What sum must be lent at simple interest, at 4 per cent. per annum, that the amount, at the end of 2 years 10 months, may be £627 18*s.* 6*d.*? Ans. £564 0*s.* 1*d.*

3. What sum must be lent at simple interest, at 6 per cent. per annum, that the amount, at the end of 18$\frac{1}{2}$ years, may be $10000? Ans. $4739.33.6+.

PROBLEM V.—*To find the time in which, at a given rate per cent. per annum, a given principal would produce a given interest.*

146. RULE.—As $\left.\begin{array}{r}\text{principal} : \$100 \\ \text{rate} : \text{interest}\end{array}\right\} :: 1 \text{ year}$

: time required.

Exercises.—1. In what time will $460 amount to $500, at 4$\frac{1}{2}$ per cent. per annum? Ans. 1 year 340 days.

2. How long must $2000 be lent at simple interest, at 3$\frac{1}{2}$ per cent. per annum, to amount to $2280? Ans. 4 years.

3. How long must $750 75 be lent at simple interest, at 7 per cent. per annum, to amount to $1013 51 2$\frac{1}{2}$.

Ans. 5 years.

PROBLEM VI.—*To find at what rate a given principal would gain a given interest in a given time.*

147. RULE.—As $\left.\begin{array}{r}\text{given time} : 1 \text{ year} \\ \text{principal} : \$100\end{array}\right\} :: \text{interest}$

: rate.

Exercises.—1. At what rate per cent. per annum, simple interest, will $1500 amount to $1850, in 4$\frac{1}{2}$ years?

Ans. $5 18$\frac{1}{2}$.

2. If a merchant, with a capital of $5000, gain $2000 in $2\frac{3}{4}$ years, at what rate per cent. per annum, simple interest, has he gained? Ans. 14.54\frac{1}{2}$ per cent.

3. At what rate per cent. per annum, simple interest, will a merchant double his capital, which is $10000, in 10 years? Ans. 10 per cent.

Questions.

What is interest?

What is called the principal?

What is meant by the rate of interest?

How many kinds of interest are there?

What is simple interest?

What is compound interest?

How is the interest of a given sum found for one year at a given rate per cent.?

Repeat the rule for finding the interest of a given principal for any other time than a year?

Repeat the rule for finding the principal, which, in a given time, would produce a given interest, at a given rate per cent. per annum.

Promiscuous Exercises in Simple Interest.

1. Required the interest of $25.75, for 1 year, at 7 per cent. per annum. Ans. 1.80\frac{1}{4}$.

2. Required the interest of $300, for 2 years, at 7 per cent. per annum. Ans. $42.

3. Required the interest of $632.25, for $2\frac{1}{2}$ years, at 6 per cent. per annum. Ans. $94.83.7$\frac{1}{2}$.

4. Required the interest of $725.50, for 2 years, at 6 per cent. per annum. Ans. $87.06.

5. What is the interest of $750, for 13 weeks, at 6 per cent. per annum? Ans. $11 25.

6. Required the amount of $500, for 5 months, at 7 per cent. per annum. Ans. $514·58.3.

7. What is the interest of $100, for 26 days, at 5 per cent. per annum? Ans. 35 cents 6+ mills.

8. How much money must be lent on the 21st of May, 1827, at 6 per cent. per annum, to bring in for interest $1500 on the 18th of January, 1832? Ans. $5352.32

9. What principal, lent on the 4th of July, 1827, at $5\frac{3}{4}$ per cent. per annum, would amount to $1250 75, on the 17th of March, 1830? Ans. 1102.05.

10. How long must \$6000.50 be lent, at simple interest, at $5\frac{1}{2}$ per cent. per annum, to amount to \$$7433.87\frac{1}{2}$?

Ans. 4 years 283+ days.

11. At what rate per cent. per annum, simple interest, will \$25000 amount to \$40000 in 10 years? Ans. 7 per cent.

Example.—A merchant takes at interest \$2500 at 7 per cent. per annum, for 2 years, with condition to pay before the time as much of the principal as he pleases: now, at the expiration of 9 months, he pays \$800, and 6 months after \$700, leaving the rest the full time of the aforesaid 2 years. How much has he then to pay?

$$\left.\begin{array}{r} \text{As } \$100 : \$2500 \\ 12 \text{ months} : 9 \text{ months} \end{array}\right\} :: \$7 : \$131\ 25$$

$$\begin{array}{r} \text{From } 2500 \\ \text{Take } 800 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \text{As } \$100 : \$1700 \\ 12 \text{ months} : 6 \text{ months} \end{array}\right\} :: \$7 : \$59\ 50$$

$$\begin{array}{r} \text{From } 1700 \\ \text{Take } 700 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \text{As } \$100 : \$1000 \\ 12 \text{ months} : 9 \text{ months} \end{array}\right\} :: \$7 : \$52\ 50$$

$$\begin{array}{rr} & \\ \hline \text{Interest} & 243\ 25 \\ \text{Principal} & 1000 \\ \hline \text{Ans. } & \$1243\ 25 \\ \hline \end{array}$$

148. The interest, in this and similar examples, may be more readily found and of course the calculation very much abbreviated by the following method: *Multiply the principals by the times, respectively, and multiply also* 100 *and* 1 *year in the same time as the given times; then, as this product is to the rate per cent. per annum, so is the sum of the above-mentioned products to the interest sought.** Let us take, for instance, the last example.

* This method of calculation will appear evident from this consideration, that the interest of \$22500 for 1 month is equivalent to the interest of \$2500 for 9 months; the interest of \$10200 for 1 month, equivalent to the interest of \$1700 for 6 months, &c.: and the interest of \$1200 for 1 month, equivalent to the interest of \$100 for 12 months.

From	\$2500 × 9 months		=22500
Take	800		
From	\$1700 × 6		=10200
Take	700		
	\$1000 × 9		= 9000

As \$1200 : \$7 :: \$41700

7

12|00)2919|00

Interest \$243·25

12. I purchased a house for \$10000; one-fourth of the purchase money was paid on the day of sale, and the rest was to remain in my hands for 18 months, at 6 per cent. per annum, simple interest, with condition to pay as much of the remainder as I pleased before the time. Now, after 3 months time, I pay \$600, and 4 months after that \$1000, and 5 months after that I pay \$750. I want to know what I have yet to pay, at the expiration of 18 months.

Ans. \$5702.50.

13. Given at interest \$6000, on the 13th of May, 1826, for 1 year, at the rate of 7 per cent. per annum, with condition that the receiver may discharge as much of the principal as he pleases before the time. Now, he pays on the 9th of July \$2000, and on the 17th of September \$1500. How much has he to pay for principal and interest at the expiration of the year? Ans. \$2733.39$\frac{43}{73}$.

149. The calculation of interest on *accounts current* affords a useful application of the preceding principles. An *account current* contains a statement of the mercantile transactions of one person with another, when immediate payments are not made.

Example.—Required the principal and interest due on the following account current, till the 10th of December, at 7 per cent. per annum.

Hence, the sum of the products of the principals and times, respectively, is put in the third place; and \$1200 and \$7 must be put in the first and second places respectively.

Mr. James Carroll, Baltimore, in Account Current, with Dennis H. Doyle, New-York.

Dr. **Cr.**

1827.		$ cts.	1827.		$ cts.
Feb. 11	To Balance, by account furnished	186 50	Mar. 24	By amount of Flour	166 50
26	To amount of Sugar	214 75	April 6	By Cash	347 75
June 20	To amount of Rum and Sugar	515 25	Sep. 26	By Bill on Abram Bell & Co.	250 25
Dec. 10	*To interest due on this account*	11 32	Dec. 10	*By Balance to your Debit in a new account*	163 32
		$927 82			$927 82

		Days.	Dr.	Cr.
Feb. 11,	To	$186·50×303*	=5650950	
26,	To	$214·75×288	=6184800	
June 20,	To	$515·25×173	=8913825	
Mar. 24,	By	$166·50×261	= - - - -	4345650
April 6,	By	$347·75×248	= - - - -	8624200
Sep. 26,	By	$250·25× 75	= - - - -	1876875
		From	20749575	14846725
		Take	14846725	
			5902850	

* For rightly understanding this calculation, it is necessary to consider that the account is made up till the 10th of December, and the interest calculated on it till that date. We place in a column, as above, all the sums on the debit side, and then all those on the credit side, prefixing to both their dates, and to the former the word *to*, and to the latter *by*, for the sake of distinction. We next find, successively, the number of days that elapse between Feb. 11 and Dec. 10, between Feb. 26 and Dec. 10, &c. and place them in the next column. A debit column and a credit one are thus formed, and all the sums on the debit side are multiplied by the corresponding number of days, and the products are placed in the debit column. In like manner, the products of the sums on the credit side by the days which follow them, are placed in the credit column. The sums of the two columns are then taken, and the debit side is found, by subtraction, to exceed the other side by $59028·50, which by means of the method pointed out in § 123, gives $11·32, the interest due on the entire account. This is placed on the debit side of the account; and then the sum of all on the credit side is taken from the sum of all on the debit side, and the remainder $163.32 is placed on the credit side, and is the sum due by the person to whom the account is furnished. It is scarcely necessary to say that the *last two* lines, in *Italics*, form the *answer* of the account, being found by the calculator. In calculating the interest on the debit side, it is the custom in some places to include both days, as in the above example.

As $36500 : $7 : : $59028·50
7

```
36500)413199·50($11·32
      36500
      -----
       48199
       36500
       ------
       116995
       109500
       ------
         74950
         73000
         -----
          1950
```

14. Required the principal and interest due on the following account current, till the 18th of January, 1828, at 6 per cent. per annum.

Ans. Interest $10.70, Principal $1296.57½.

Dr. Mr. J. Fox, in Account Current with A. Bell. Cr.

		$	cts.			$	cts.
1827.				1827.			
May 19	To Goods	512	37½	June 13	By Cash	400	50
Aug. 23	To Tea	273	50	Nov. 8	By Linen	680	00
Oct. 4	To Goods	200	00	Dec. 1	By Bill	81	50
Nov. 18	To Sugar	300	00	1828.			
1828.				Jan. 18	*By Balance to new Account*	134	57½
Jan. 18	*To Balance of Interest*	10	70				
		$1296	57½			$1296	57½

15. Required the principal and interest due on the following account current, till the 26th of June, 1828, at 5 per cent. per annum.

Mr. Joseph Foulks, New-Orleans, in Account Current with John Moreau, New-York.

Dr.		$ cts.		Cr.	$ cts.
1827.			1827.		
Sep. 3	To Balance	2000 00	Sep. 19	By Sugar	1675 50
Dec. 21	To Flour	765 75	1828.		
1828.			Mar. 20	By Tobacco	1236 75
Jan. 27	To Goods	1040 50	May 25	By Cotton	912 25
Mar. 26	To Linen	838 75	June 26	*By Balance to new Account*	868 59
June 26	*To Balance of Interest*	48 09			
		$4693 09			$4693 09

16. On the 5th of January, 1822, the public funded debt of Great Britain and Ireland consisted of

£	s.	d.
534355086	6	11, at 3 per cent. per annum.
29547003	19	3, at $3\frac{1}{2}$
75947763	19	4, at 4
and 153056763	7	9, at 5

Required the annual interest of the whole.

Ans. £27755546 9*s.* $1\frac{1}{2}$*d.*

Discount.

150. *Discount* is an abatement made for advancing money before it becomes due. The money which is received as the full payment of any debt or bill due some time after, is called its *present worth.*

Problem I.—*To find the present worth of a bill or debt.*

151. Rule.—Find the interest of the debt at the given rate, and for the given time; consider this interest as discount, and subtract it from the debt to find the present worth.*

* This rule for the calculation of discount is that which is always employed by merchants and bankers. It is founded, however, on a principle radically false; and always *gives the discount too large, and consequently the present worth too small*, by the *interest of the true discount.* This will appear manifest, if we consider that *the true present worth of any debt is such a sum as would, if lent at interest at the assigned rate, amount to that debt at the time at which it would have been due*: and consequently the discount, or the difference between the present worth and

Example 1. Required the present worth of a bill of $350, due at the end of 3 months, at 6 per cent. per annum.

Here, by the method already explained in interest, the discount is readily found to be $5 25, and this being taken from $350, the remainder, $344 75, is the present worth.

2. What is the present worth of a bill of £39 5s., due on the 1st of September, but paid on the 3d of July preceding, discount being allowed at 5 per cent. per annum?

The time here is 60 days, for which the interest of £39 5s. is found to be 6s. 5$\frac{7}{73}$*d.*; and by subtracting this from £39 5s. we have remaining £38 18s. 6$\frac{42}{73}$*d.*, the present worth.*

3. Required the present worth of a bill of $346, drawn 8th of March, at 6 months, and discounted 3d of June, at 6 per cent. per annum.

By counting forward 6 months and 3 days, from the 8th of March, we find this bill to be due on the 11th of September. The number of days from the 3d of June till this date is 100, and the interest of $346 for 100 days, at 6 per cent. per annum, is found, by any of the methods formerly explained, to be $5.69; and, consequently, the present worth is $340.31.

It is proper to observe, that in each of the following exercises, 3 days of grace must be allowed.

Exercises.—1. Required the present worth of a bill of

the debt, should be, not the interest of the debt, but the interest of the present worth; and therefore the interest of the debt will exceed the true discount, that is, the interest of the present worth, by the interest of that discount. The *true* present worth will be found by the rule to Problem II. of the present article.

* In the United States, as well as in Great Britain and Ireland, three days, called *Days of Grace*, are always allowed after the time a bill is nominally due, before it is legally due: thus, suppose a bill were drawn on the 8th of April, at 4 months, it would be due, not on the 8th, but on the 11th of August.

It may be remarked, that if, without the days of grace, a bill should appear to be due on the 31st of a month which contains only 30 days, the last day of that month is to be taken, and not the 1st of the next; and, consequently, the 3d of the next month will be the day on which, by the addition of the days of grace, the bill will be really due. Thus, a bill drawn on the 31st of August, at 3 months, would be due on the 3d of December. In like manner, a bill which, without the addition of the days of grace, would be due on the 29th, 30th, or 31st of February, if that month contained so many days, would be really due on the 3d of March. It may be farther remarked, that bills which fall due on Sunday, are paid, in the United States as well as in England, on Saturday, but in Ireland on Monday.

$500, drawn 1st of March, at 7 months, and discounted June 9th, at 6 per cent. per annum. Ans. $489.72.

2. Required the discount of $285, for 1 year, at 6 per cent. per annum. Ans. $17.24.

3. What is the present worth of a bill of $250, payable in 60 days, at 6 per cent. per annum? Ans. $247.41.

4. What is the discount of $675.75, payable in 90 days, at 6 per cent. per annum? Ans. $10.33.

5. What is the present worth of $175.50, payable in 30 days, at 7 per cent. per annum? Ans. $174.39.

6. Required the present worth of a bill of $500, drawn on the 20th of May, and due on the 4th of October, at 5 per cent. per annum. Ans. $489.72.

7. Required the present worth of a bill of $875.75, drawn on the 8th of September, at 5 months, and discounted on the 12th of November, at 6 per cent. per annum. Ans. $862.64.

8. Required the present worth of a bill of $670, drawn on the 8th of June, at 90 days, and discounted on the 10th of July, at 6 per cent. per annum. Ans. $663.28.

9. Required the discount of a bill of $2000, drawn on the 10th of April, at 60 days, and discounted on the 1st of May, at 6 per cent. per annum. Ans. $1986.19.

10. What is the present worth of $196.75, due at the end of 9 months, at 6 per cent. per annum. Ans. $187.80.

11. What is the present worth of a bill of $1400, drawn the 1st of August, at 18 months, and discounted on the 25th of December, at 6 per cent. per annum? Ans. $1306.56.

PROBLEM II.—*To find the true present worth of a bill or debt.*

152. RULE.—As the amount of $100 for the given time and proposed rate, is to $100, so is the debt to its true present worth: and the present worth being subtracted from the debt, the remainder is the discount.

Example 1.—Required the true present worth of $210, due at the end of a year, at 5 per cent. per annum.

In this case, the amount of $100 being $105, we have, by the rule, this analogy: As $105 : $100 :: $210 : $200, the true present worth required.

```
As $105 : $100 : : $210 :
                     100
                   -----
           105)21000($200
               210
               ---
```

By the common rule, the result would be \$199.50, consequently the error is 50 cents.*

Ex. 2. Required the true present worth of \$478.40, for 8 months, at 6 per cent. per annum.

Here, the amount of \$100 is \$104; and therefore, as \$104 : \$100 :: \$478.40 : \$460. By the common method, the result would be \$459.40 nearly; and the error 60 cents.

This question, and all similar ones, may be very easily wrought, by the following rule:—*Multiply the months by the rate, and add the product to* 1200; *then, as the sum is to* 1200, *so is the debt to its true present worth.* Thus, in the preceding example, we should have this analogy, 1248 : 1200 :: \$478.40 : \$460.†

3. Required the correct present worth of \$730, due on the 19th of September, but paid on the 8th of May preceding, at 6 per cent. per annum.

Here, the number of days being 134, we have this analogy: as 365 days : 134 days :: \$6 : \$2·20 nearly; the interest of \$100 for 134 days; and then, as \$102·20 : 100 :: \$730 : \$714·28½, answer.

This question, and all others in which it is required to find the *correct* present worth of a debt, for a given number of days, may be wrought more easily and more accurately by the following rule:—*Multiply the days by the rate, and add the product to* 36500 (=365×100) *then, as this sum is to* 36500, *so is the debt to its true present worth:* thus, in the present example, 134×6=804, and 36500+804 37304: then, as 37304 : 36500 :: \$730 : \$714·28½, the present worth.‡

* *The reason of this rule* will be evident from the consideration, that \$100 is the present worth of its amount regarded as a debt: and consequently the analogy given above will be simply this: as the amount of \$100 considered as a debt, is to \$100, the present worth of that debt, so is any other debt to its present worth. It is obvious, also, that for the first two terms of the analogy, we might use the amount of *any sum whatever*, and that sum itself; but it is generally more simple and easy to employ \$100 and its amount.

† The reason of this rule may be thus shown: as 12 months : 8 months :: \$6 : \$$\frac{48}{12}$, the interest of \$100 for 8 months. Then, as \$100$\frac{48}{12}$: \$100, or by reduction of both to twelfths, as 1248 : 1200 :: \$478.40 : \$460.

‡ This rule depends on the same principle as the last; and the reason of it may be thus illustrated. As 365 days : 134 days :: \$6 : \$1$\frac{74}{365}$, the interest of \$100 for 134 days, at 6 per cent. per annum. Then, as \$102$\frac{74}{365}$: \$100; or by reduction to three hundred and sixty-fifths, as 37304 : 36500 :: the debt to its present worth. Both these rules are in reality the same as rule IV. in Interest. From the above

Should it be thought advisable for the learner to work discount in the correct method, the exercises at the beginning of this article will serve his purpose as well as any others; and the following are their answers by that method:—

True Answers.

Ex. 1	\$489.94.	Ex. 4	\$10.17½.	Ex. 7	\$862.84.3
2	\$16.26.	5	\$174.40.	8	\$663.37.
3	\$247.44.	6	\$489.93.	9	\$1986.28½

It appears evident, by comparing these answers with those found by the common method, § 151, that when the time is short, as it generally is in real business, the difference between the results found by the two methods, is inconsiderable; and therefore, the common method, the calculation for which is so easy, may be employed without much error. Still, however, the principle is false, as it gives profits to the discounter, which are not proportional to the times. It may be said, indeed, that those who keep money for the purpose of discounting, are entitled to more than the simple

examples it will appear how very erroneous the common method of computing discount is, especially when the time is long. In every case, in fact, the discounter of the bill has a greater rate of interest for his money, than the nominal rate; and the longer the time, the greater is this rate. Thus, if a person have a bill for \$100, payable at the end of a year, at 5 per cent., he will receive, according to the common method of discount, only \$95 for it; and were he to lend this sum for a year at the same rate, instead of \$100, to which it obviously should amount, he would receive only \$99.75. The true present worth is \$95.23$\frac{17}{20}$, and consequently the error 23$\frac{17}{20}$ cents. Again, had the bill been payable in 2 years, the present worth, by the common method, would have been \$90, while it should be \$90.90$\frac{10}{11}$. The error is consequently 90$\frac{10}{11}$ cents, and \$90, instead of amounting to \$100, at the end of two years, would amount to no more than \$99. Had the times been 4 years, the present worths would be \$80, and \$83$\frac{1}{3}$, and the error \$3.33$\frac{1}{3}$. The amount also of the present worth \$80, at the end of 4 years would be \$96, and consequently \$4 less than it should be. If the time had been 10 years, the present worths would have been \$50 and \$66$\frac{2}{3}$, where the error is \$16$\frac{2}{3}$; and the amount of the present worth \$50, at the end of 10 years, would be \$75 instead of \$100. Finally, were the time 20 years, the present worth, according to the common method, would be *nothing*, while it should be \$50: and were the time greater than 20 years, the present worth would be unassignable, as it would appear to be less than nothing; or if any meaning could be attached to the result of the operation, it would be that the person who held the bill, instead of receiving any thing for it, would be required to pay something to get it off his hands.

S 2

common rate. This may be true; yet at the same time, it does not prove the correctness of the principle on which this mode of computation depends; since, if the discounter is to have a greater rate, it should be some fixed rate, and not a variable one, depending on the time which a bill has to run.

Questions.

What is discount?

What is the present worth of a bill or debt?

Repeat the rule, employed by merchants and bankers, for calculating discount.

Repeat the rule for finding the true present worth of a bill or debt.

Commission, Insurance, &c.

153. *Commission* is the sum which a merchant charges for buying or selling goods for another.

154. *Brokerage*, is a smaller allowance of the same nature, paid usually for negotiating bills, or transacting other money concerns.

155. *Insurance* or *Assurance* is a contract by which one party, on being paid a certain sum or premium by another, on account of property that is exposed to risk, engages in case of loss, to pay to the owner of the property the sum insured on it.

Problem I.—*To compute the commission, brokerage, insurance, or any other allowance, on a given sum, at a given rate per cent.*

156. Rule.—Multiply the sum by the rate per cent. and divide the product by 100: or, as \$100 : to the rate per cent. : : the given sum : the allowance required.

Example 1. Find the commission on \$750·75, at $2\frac{1}{2}$ per cent. As \$100 : \$$2\frac{1}{2}$: : \$750·75 : \$18·77.

Here, by multiplying by $2\frac{1}{2}$, and dividing by 100, we find for the answer \$18·$76\frac{7}{8}$, or \$18·77 nearly. The same result would be obtained by dividing the given sum by 40, since $2\frac{1}{2}$ is a fortieth of 100.

2. Find the brokerage on \$1872, at $\frac{3}{4}$ per cent.

As \$100 : \$$\frac{3}{4}$: : \$1872 : \$14·04.

Here, by multiplying 1872 by $\frac{3}{4}$, and dividing the product by 100, we find for the answer \$14·04.

3. Required the premium of insurance on \$512·50, at 6 per cent. As \$100 : \$6 : : \$512·50 : \$30·75.

Here, by multiplying \$512·50 by 6, and dividing the product by 100, we find for the answer 3075 cents, or \$30·75.

PROBLEM II.—*To find how much must be insured on property worth a given sum, so that in case of loss, both the value of the property and the premium of insurance may be repaid.*

157. RULE.—Subtract the rate from \$100; then, as the remainder is to \$100, so is the value of the property to the sum to be insured.

Example 4. How much must be insured at 8 per cent. on an adventure of \$6440, that in case of loss, not only the value of the adventure, but also the premium of insurance may be paid?

Here as \$92 (=100—8) : \$100 : : \$6440 : \$7000. The truth of this operation is proved by finding the premium on \$7000 at 8 per cent. This is found to be \$560. Hence, in case of the adventure being lost, the owner will receive not only \$6440, the value of the adventure, but also \$560 the premium; and thus he will sustain no loss whatever.*

Exercises.—1. What is the commission on \$735, at $4\frac{1}{2}$ per cent.? Ans. \$33·07·5.

2. What is the brokerage on \$1569, at $\frac{7}{8}$ per cent.? Ans. \$13·72·8+.

3. What is the premium of insurance on \$1750·75, at $5\frac{3}{4}$ per cent.? Ans. \$100·66·8+.

4. What is the expense of insuring a vessel and cargo, worth \$12756·50, at $3\frac{1}{4}$ per cent.? Ans. \$414·58·6.

5. What must be the sum insured, at $5\frac{1}{4}$ per cent. on goods worth \$4967·62$\frac{1}{2}$, so that in case of loss, the owner may be repaid both the value of the goods and the premium of insurance? Ans. \$5242·87$\frac{1}{2}$.

6. Add to \$535.25, the commission on itself at $4\frac{1}{2}$ per cent. and find the insurance of the sum at $4\frac{3}{8}$ per cent. Ans. \$26·57.

* The reason will appear manifest, by considering, that in receiving 100 dollars which had been insured at 8 per cent., the owner would receive but 92 dollars in lieu of the property, 8 dollars being paid for the insurance.

7. At $3\frac{3}{4}$ per cent. how much must be insured on goods worth $1635·50, so that in case of loss, the owner may be entitled to the value of the goods, and the premium?

Ans.

8. Invoice* of flour, shipped by Abram Bell & Co. on board the ship James Cropper, H. Graham, master, for Liverpool, by order, and for account and risk of George Creswell, Merchant, London.

			$	cts.
A. B.	2000 barrels of Flour, at $5·25 per barrel,		10500	
	CHARGES.			
	Paid carriage and shipping, cooperage, bills of lading, &c.	$250.37½		
	Insurance, at $2\frac{3}{4}$ per cent.	295·63½		
	Commission, at $3\frac{1}{2}$ per cent.	386·51		
			932	52
	Errors excepted.			
	New-York, April 23d, 1827.		11432	52
	ABRAM BELL & CO.			

Questions.

What is commission?
What is brokerage?
What is insurance?

Repeat the rule for calculating the commission, brokerage, &c. on a given sum, at a given rate per cent.

Repeat the rule for finding how much must be insured on property, so that in case of loss, both the value of the property and the premium of insurance may be repaid.

* An invoice is an account containing the quantity, prime cost, and charges of goods sent from one person to another, usually by sea. In this invoice, the letters (A. B.) in the margin, are the letters with which the barrels were marked. The two lines in italics, form the *answer* of the exercise, being found by the calculator.

CHAPTER IX.

Barter; and Profit and Loss.

BARTER.

158. *Barter* is the exchanging of one commodity for another; value for value, according to rates or prices agreed upon by the parties concerned.

159. RULE.*—Find the value of that commodity whose quantity is given; then find what quantity of the other, at the rate proposed, you may have for the same money, and it gives the answer required.

Example. How much coffee at $16\frac{1}{2}$ cents per lb. must be given in barter for 1 pipe of wine, at \$1·37½ per gallon?

Here the price of the wine is found by the first analogy to be \$173·25, and the number of pounds of coffee is found, by the second analogy, to be 1050 lbs., so that 1 pipe of wine, containing 126 gallons, at \$1·37½, is equal in value to 1050 lbs. of coffee, at 16½ cents per lb.

As 1 gal. : 126 gal. :: \$1·37½ : \$173·25.

cts. *cts.* *lb.* *lbs.*

As 16½ : 17325 : : 1 : 1050.

Exercises.—1. How many yards of linen, at 87½ cents per yard, must be given in barter for 6 pieces of black cloth, each containing 37*yds.* 2*qrs.* at \$5·75 per yard?

Ans. 1478*yds.* 2*qrs.* 1*n.*

2. A. delivered 5 pipes of brandy, at \$1·18¾ per gallon, to B. for 2497 gallons of rye whiskey; what was the whiskey per gallon? Ans. 29cts. 9m.+.

3. How much wheat, at \$1·12½ per bushel, must be given in barter for 125 barrels of flour, at \$5·75 per barrel?

Ans. 638⅘ bush.

4. How much barley, at 62½ cents per bushel, must be given in barter for 12 barrels of beer, at \$6·25 per barrel?

Ans. 120 bush.

5. A. and B. barter; A. has 25 barrels of cider, at \$4·50 per barrel, for which B. gave him 76 lbs. of tea, at 94 cents

* This rule is, evidently, only an application of the rule of Simple Proportion.

per lb. and the rest in sugar, at $10\frac{1}{2}$ cents per lb.; how much sugar must B. give A. besides the tea? Ans. $483\frac{1}{2}\frac{9}{11}$*lbs.*

6. A. has 3 hhds. of sugar, weight neat 27*cwt.* 3*qrs.* 14*lbs.* at \$9·50 per cwt., for which he receives from B. in barter, 2 chests of tea, each containing 95 lbs. at \$1·09 per lb. and the rest in money; how much money did A. receive from B.? Ans. \$56·83¾.

Profit and Loss.

160. That branch of Arithmetic which treats of the gains or losses on mercantile transactions, is called *profit and loss*; and it teaches merchants how to fix the price of their goods so as to gain so much per cent.*

Questions in this rule are performed by simple proportion, upon this principle, that quantities, or sums of money, which gain or lose at the same rate, are to one another as their gains or losses. The method of performing the first example and the first six exercises, is so obvious as not to require a formal rule.

Example. If 1 cwt. of sugar be bought for \$10·50, and sold at 11 cents per lb. what is the gain?

Here, the hundred weight being bought for \$10·50, and sold for \$12·32, the gain is evidently \$1·82, the difference between them.

As 1lb. : 112lbs. : : 11cts. :
11

Selling price \$12·32
First cost 10·50

Gain 1·82 Ans.

Exercises.—1. If a piece of cloth, containing 31*yds.* 2*qrs.* cost \$187, what is gained by selling it at \$7·25 per yard? Ans. \$40·87½.

2. Bought 212*cwt.* 1*qr.* 14*lb.* of pork, at \$4·25 per cwt. and the charges amount to \$75·75; what is the gain or loss, by selling it at 6¼ cents per lb.? Ans. Gain \$507·27½.

3. If a pipe of wine, containing 130 gallons, cost \$250·50, what is gained by selling it at \$2·12½ per gallon? Ans. \$25·75.

* It should be particularlv remarked, that *by the gain or loss per cent. is to be understood the sum that would be gained or lost at the given rates,* not on a hundred dollars worth sold, *but on a hundred dollars laid out in prime cost, and in charges, if there be any.*

4. If a chest of tea, containing 110 lbs. be sold at \$1·13 per lb., what is the gain, the first cost being \$96·75?

Ans. \$27·55.

5. If a cwt. of pork be bought for \$6·25, and sold at $6\frac{1}{4}$ cents per lb., what is the gain? Ans. \$0·75.

6. If a barrel of cider, containing $31\frac{1}{2}$ gallons, be bought for \$4·25, what is gained by selling it at $6\frac{1}{4}$ cents a pint?

Ans. \$11·50.

PROBLEM I.—*From the prime cost and selling price, to find the gain or loss per cent.*

161. RULE.—As the prime cost : to the gain or loss on the cost : : \$100 : the gain or loss per cent.

Exam. 2. If tea be bought at $87\frac{1}{2}$ cents per lb. and sold at \$1·$12\frac{1}{2}$ per lb., what is the gain per cent.?

From \$1·$12\frac{1}{2}$
Take ·$87\frac{1}{2}$

As $87\frac{1}{2}$ cts. : 25 cts. : : \$100 :

Here, \$1·$12\frac{1}{2}$—·$87\frac{1}{2}$=25 cents, the gain on $87\frac{1}{2}$ cents; then, as $87\frac{1}{2}$ cents (the first cost of 1 lb.) : 25 cents, (the gain on $87\frac{1}{2}$ cents) : : \$100 (regard as first cost) : \$28·$57\frac{1}{7}$, the gain per cent., or the gain on \$100.

Exam. 3. If coffee be bought at $16\frac{3}{4}$ cents per lb., and sold at $16\frac{1}{2}$ cents per lb., what is the loss per cent.?

Here, subtracting $16\frac{1}{2}$ from $16\frac{3}{4}$, the remainder is $\frac{1}{4}$ cent, which is the loss on $16\frac{3}{4}$ cents; therefore, as $16\frac{3}{4}$: $\frac{1}{4}$: : \$100 : \$1·$49\frac{1}{4}$, nearly, the loss per cent., or the loss on 100 dollars.

Ex.—7. If wine, which cost \$1·$37\frac{1}{2}$ per gallon, be sold for \$1·$62\frac{1}{2}$, what is gained per cent.? Ans. \$18·18·$1\frac{2}{11}$.

8. If paper, which cost \$4·25 per ream, be sold for \$3·95 per ream, how much is lost per cent.? Ans. \$7·05·$8\frac{14}{17}$

9. If sugar cost \$9·$37\frac{1}{2}$ per cwt., and sold at $11\frac{1}{2}$ cents per lb., what is gained per cent.? Ans. \$37·$38\frac{16}{25}$.

10. Bought tea at 94 cents per lb., and sold it at \$1·10 per lb., how much is the gain per cent.? Ans. \$17·02·$1\frac{1}{4}\frac{3}{7}$.

11. Bought Newton's Principia for \$10·80, and sold it for \$10, how much is the loss per cent.? Ans. \$7·40·$7\frac{11}{27}$.

12. Bought Gregory's Mechanics for \$11·50, and sold it for \$14, how much is the gain per cent.? Ans. \$21·73·$9\frac{3}{23}$.

13. Bought Wheewell's Mechanics and Dynamics for \$7·25, Laplace's System of the World for \$5·75, Bonnycastle's Algebra for \$6·$37\frac{1}{2}$, and Simpson's Fluxions for \$5·75:

sold all these books for $30; how much is the gain per cent.? Ans. $19·40$\frac{8}{9}$.

Problem II.—*To find how a commodity must be sold to gain or lose a certain rate per cent.*

162. Rule.—As $100 : the gain or loss per cent. : : the prime cost : the gain or loss on that cost; and from this and the prime cost, the selling price will be found by addition or subtraction.

Exam. 4. How must tea, which cost 92 cents per lb. be sold to gain 25 per cent.?

Here, as $100 (regarded as first cost) : 325 (the gain on $100) : : 92 cents (the first cost of 1 lb.) : 23 cents, the gain per lb. which being added to 92 cents, the amount is $1·15, the required rate.

As $100 : $25 : : 92 cents :
4 1
Gain 23 cts.
Selling price $1·15 Ans.

5. Sold a pipe of wine, which cost $1·75 per gallon, at a loss of 12$\frac{1}{2}$ per cent.; at what rate per gallon was it sold?

Here, the loss on $1·75, is found to be 21$\frac{7}{8}$ cents, which, taken from $1·75, the first cost, gives the rate required.

As $100 : 12\frac{1}{2}$: : $1·75 : 21$\frac{7}{8}$ cts.
From $1·75 cost
Take ·21$\frac{7}{8}$ loss
$1·53$\frac{1}{8}$ sales.

Ex. 14. How must wine, which cost $1·27 per gallon, be sold to gain 20 per cent.? Ans. $1·52·4.

15. How must linen, which cost 56 cents per yard, be sold to gain 30 per cent.? Ans. $0·72·8.

16. How must superfine black cloth, which cost $9·75 per yard, be sold to gain 22 per cent.? Ans. $11·89$\frac{1}{2}$.

17. Sold a piece of linen, which cost 75 cents per yard, at a loss of 20 per cent., at what rate per yard was it sold? Ans. $0·60.

18. Bought a book for $14·50, and sold it at a loss of 12$\frac{1}{2}$ per cent. For how much was the book sold? Ans. $12 68$\frac{3}{4}$.

19. How must pork, which cost 4$\frac{1}{2}$ cents per lb., be sold to gain 20 per cent.? Ans. $0·05·4.

20. Bought beef at 3$\frac{3}{4}$ cents per lb. How must I sell it to gain 37$\frac{1}{2}$ per cent.? Ans. $0·05·1$\frac{9}{16}$.

Problem III.—*From the gain per cent, and the selling price, to find the first cost.*

163. Rule.—As $100, together with the gain per cent., or diminished by the loss per cent. : $100 : : the selling price to the prime cost.

Exam. 6. What was the first cost of brandy, which being sold at $1·15 per gallon, the seller clears 20 per cent.?

In this example, it is evident, that what cost $100 is sold for $120; and the analogy is no more than this: as the selling price, $120, is to its first cost, $100, so is the selling price, $1·15, to its corresponding first cost, $95\frac{5}{6}$ cents.

$$\begin{array}{rrr} \text{As } \$120 & : \$100 & : : \$1{\cdot}15 \\ \hline 6 & 5 & 5 \\ \hline & & 6)5{\cdot}75 \\ \hline & & \text{Ans. } {\cdot}95\tfrac{5}{6} \text{ cts.} \end{array}$$

Exam. 7. If a merchant, by selling wine at $2·25 per gallon, lose 18 per cent., what was the prime cost?

As $82 (=100—18) : $100 : : $2·25 : $2·74$\frac{16}{41}$.

In this example, it is evident, that what cost $100 must have been sold for $82; and therefore the preceding analogy is simply this: as the selling price, $82, is to the corresponding first cost, $100, so is the selling price, $2·25, to the corresponding first cost, $2·74$\frac{16}{41}$, or $2.75, nearly.

Ex. 21. If 12 per cent. be lost by selling 128 yards of broadcloth for $500, what was the prime cost per yard?

Ans. 568\frac{2}{11}$.

22. Suppose a book to be sold for $35·50, and 20 per cent. to be gained, what was the prime cost? Ans. $29·58$\frac{1}{3}$.

23. If 15 per cent. be gained by selling tea at $1·12$\frac{1}{2}$ per lb., what was the first cost, and what was gained by the sale of 350 lbs.? Ans. 97$\frac{19}{23}$ cents first cost, $51·36 gain.

Exam. 8. If 20 per cent. be gained by selling wine at $2·12$\frac{1}{2}$ per gallon, how must it be sold to gain 25 per cent.?

This example might be wrought by finding the first cost, and thence the selling prices, as in the preceding exercises. It will be wrought more easily, however, by this analogy; as $120 : $125 : : $2·12$\frac{1}{2}$: the price required. The reason of this will be manifest, if it be considered, that what cost $100, in the one case sold for $120, and in the other for $125; and therefore, the above analogy is no more than this: as the selling price of one hundred dollars worth in

the one case, is to its selling price in the other, so is the selling price of a gallon in the first case, to its selling price in the other.

Ex. 24. If 15 per cent. be gained by selling sugar at \$8·75 per cwt., how much is gained per cent. by selling it at \$9·25? Ans. $21\frac{402}{691}$ per cent. or \$21·58·2 per 100 dolls.

25. If 6 per cent. be gained by selling Canton crape at \$13·25 per piece, how much per cent. would be gained by selling the same at \$15·37½ per piece? Ans. 23 per cent.

26. If 20 per cent. be gained by selling flour at \$5·87½ per barrel, how much per cent. is gained or lost by selling it at \$4·25 per barrel? Ans. $13\frac{568}{2927}$ per cent.

27. Bought 21300 yards of linen, at 2*s.* 9½*d.* per yard, and paid for various charges £88 15*s.* 1*d.*; and sold one third at 3*s.*, one third at 3*s.* 4*d.*, and one third at 3*s.* 2*d.* per yard: required the whole gain, and the gain per cent.

Ans. £310 12*s.* 5*d.* and £10 2*s.* 10¾*d.*

CHAPTER X.

Division into Proportional Parts; Fellowship; and Alligation.

DIVISION INTO PROPORTIONAL PARTS.

PROBLEM.—*To divide a given quantity into parts which shall have to each other given ratios.*

164. RULE.—As the sum of the numbers expressing the ratios, is to any one of these numbers, so is the entire quantity to be divided, to the part corresponding to the number used as the second term of the analogy.*

* When all the parts, except one, have been determined, that one may be found by adding the rest together, and taking the sum from the number to be divided. In this case, the operation will be proved by finding that part by the general rule: the agreement of the two results will prove the accuracy of the work.

The reason of this rule, which is of frequent use in Arithmetic, and other branches of Mathematics, is obvious from this principle, that *the sum of all the antecedents of any number of equal ratios, is to the sum of all the consequents, as any one of the antecedents is to its consequent.*

The operation is proved by adding the several results together: if the sum be equal to the quantity to be divided, the work is right.

Example. A farm of 756 acres, 2 roods, 14 poles, is to be divided into two parts, such that the one may be three-fourths of the other. What are the parts?

Here, the parts are evidently in the ratio of 4 to 3, the sum of which is 7; therefore,

			acrs.	*r.*	*p.*		*acrs.*	*r.*	*p.*	
As 7 :	4	: :	756	2	14	:	432	1	8,	the greater part,
and as 7 :	3	: :	756	2	4	:	324	0	36,	the lesser part.

The sum of these parts is 756 2 4, which proves the work.

Exercises.—1. Suppose a traveller to proceed from New-York to Washington City, at the rate of 6 miles per hour, and another at the same time from Washington City for New-York, at the rate of 5 miles per hour: where will they meet, the distance between the two places being 228 miles?

Ans. $124\frac{4}{11}$ miles from New-York.

2. Divide 398 into three parts, which will be to one another as the numbers 5, 7, and 11.

Ans. $86\frac{12}{23}$, $121\frac{3}{23}$, and $190\frac{8}{23}$.

3. Divide \$800 into four parts, in the ratio of 10, 19, 9, and 2. Ans. 200, 380, 180, and 40.

4. Divide \$38000 among three persons, in such a manner that the share of the second may be one half greater than that of the first, and the share of the third one half greater than that of the second.* Ans. \$8000, 12000 and 18000.

5. Pure water is composed of two gases, or kinds of air, called oxygen and hydrogen, in such proportions, that the weight of the former is to that of the latter as 15 to 2. Required the weight of each contained in a cubic foot, or 1000 ounces Avoirdupois weight of water.

Ans. $882\frac{6}{17}$*oz.*, and $117\frac{11}{17}$*oz.*

6. How much copper and how much tin will be required to make a cannon, weighing 16*cwt.* 1*qr.* 20*lbs.*, gun metal being composed of 100 parts of copper, and 11 of tin?

Ans. 14*cwt.* 3*qrs.* $5\frac{73}{111}$*lbs.*; and 1*cwt.* 2*qrs.* $14\frac{38}{111}$*lbs.*

7. The British standard gold for coinage consists of 11 parts of pure gold, and 1 part of alloy, usually a mixture of

* In this exercise, it is easy to see, that the parts will be as the numbers 1, $1\frac{1}{2}$, and $2\frac{1}{4}$; or as 4, 6, and 9.

silver and copper: how much pure gold and how much alloy are contained in a guinea, which weighs 5 dwts. 9 grains?
Ans. 4*dwts.* $22\frac{1}{4}$*grs.*, and $10\frac{3}{4}$*grs.*

8. The British silver coin consists of 37 parts of silver and 3 of copper; how much of each does the half crown, worth 2*s.* 6*d.* contain, each pound, Troy weight, being coined into 66 shillings? Ans. 8*dwts.* 8·$9\frac{3}{4}$+*grs.*, and $16\frac{4}{11}$*grs.*

9. How much tin and copper are contained in a bell, weighing 150 lbs.; bell metal being composed of 3 parts of copper and 1 part of tin? Ans. $112\frac{1}{2}$*lbs.*, and $37\frac{1}{2}$*lbs.*

10. Pewter is composed of 112 parts of tin, 15 of lead, and 6 of brass; how much of each ingredient is requisite to make a ton of pewter?
Ans. 16*cwt.* 3*qrs.* $10\frac{6}{19}$*lbs.*, 2*cwt.* 1*qr.* $0\frac{12}{19}$*lb.*, and 3*qrs.* $17\frac{1}{19}$*lbs.*

11. Proof spirits are composed of 48 parts of alcohol, or pure spirit, and 52 parts of water. How much of each of these contained in 84 gallons of proof spirits?
Ans. $40\frac{8}{25}$*galls.*, and $43\frac{17}{25}$*galls.*

12. 76 parts of nitre, 14 of charcoal, and 10 of sulphur, compose gunpowder: how much of these ingredients will be requisite to form one hundred weight of gunpowder?
Ans. 3*qrs.* $1\frac{3}{25}$*lbs.*, $15\frac{17}{25}$*lbs.*, and $11\frac{1}{5}$*lbs.*

Fellowship.

165. *Fellowship* is the method of determining of the partners in a mercantile company.

166. Fellowship is usually distinguished into two kinds, *simple* and *compound*, or *single* and *double.*

167. In *simple* or *single* fellowship, the stocks or sums contributed by the several partners, all continue in trade for the same time.

168. In *compound* or *double* fellowship, the stocks continue in trade for different periods.

Simple Fellowship and Bankruptcy.

169. Rule.—As the whole stock is to the whole gain or loss, so is the stock of any partner to his gain or loss.

In the same way, the estate of a bankrupt may be divided among his creditors by this analogy—As the

sum of all the claims on the estate is to its value, so is the claim of any creditor to his dividend, or share of the estate.*

Example 1. Three merchants, A, B, and C, form a joint capital, of which A contributes \$700, B \$1000, and C \$1600. What is the share of each in a gain of \$880?

Here, the sum of the stocks is \$3300, the whole capital. Then, as \$3300 : \$880, or, by contraction, as 15 : 4 :: \$700 : \$186 66$\frac{2}{3}$, A's share; and as 15 : 4 :: \$1000 : \$266 66$\frac{2}{3}$, B's share; and, lastly, as 15 : 4 :: \$1600 : \$426 66$\frac{2}{3}$, C's share. The sum of these shares is exactly \$880, which proves the operation to be correct.

2. A bankrupt owes to A \$900, to B \$850, to C \$640, to D \$150, to E \$750, and to F \$310; but his whole estate amounts to \$1500. Required the share of each creditor.

Here, the sum of the debts is \$3600. Then, as \$3600 : \$1500, or, by contraction, as 36 : 15 : : \$900 : \$375, A's dividend; and 36 : 15 : : \$850 : \$354.16$\frac{2}{3}$, B's dividend. In the same manner, we find C's share to be \$266.66$\frac{2}{3}$; D's \$62.50; E's \$312.50; and F's \$129.16$\frac{2}{3}$. The sum of all these is \$1500, which proves the work to be correct.†

Exercises.—1. A's and B's stocks are \$375 and \$425, respectively: required the share of each in a gain of \$240.

Ans. A's \$112 50, and B's \$127 50.

2. Three merchants, A, B, and C, enter into partnership: A puts into the stock \$1555.50, B \$12591.75, and C \$1876.75. Required the share of each in a gain of \$2000.

Ans. A's \$194.14.6, B's \$1571.16.1, and C's \$234.24.2.

3. If a bankrupt, whose property amounts to £2100, owe to A £826 12*s*., to B £1263 9*s*. 6*d*., to C £724 15*s*. 10*d*., to D £1000, and to E £242 16*s*. 4*d*.; how much can he pay in the pound, and what is the dividend of each creditor?

* This rule is merely a particular application of that contained in the last article, and therefore requires no separate illustration. The method of proof is also the same.

† In the division of a bankrupt's estate, it is usual first to find *how much in* the dollar, or pound, he can pay; that is, how much the creditors will receive for each dollar, or pound, of their respective claims: thus, resuming the same example, we have this analogy—as 3600 : 1500, or as 36 : 15 : : \$1 : $\frac{15}{36}$, the sum that each creditor is to receive in the dollar. Then, as \$1 : \$$\frac{15}{36}$: : \$900 : \$375, as before. In the same way the rest of the dividends may be found.

Ans. 10*s*. $4\frac{1}{4}$*d*., nearly, in the pound; and the dividend of A £427 15*s*. 11*d*.; B £653 17*s*. $10\frac{3}{4}$*d*.; C £375 2*s*. $1\frac{1}{2}$*d*.; D £517 10*s*. $8\frac{3}{4}$*d*.; and of E £125 13*s*. 4*d*.+

4. A debtor, the value of whose effects is only \$1075, owes to A \$350.50, to B \$750.75, to C \$1270, and to D \$1875.50. What is the dividend of each and how much is paid in the dollar? Ans. A \$88.72.$3\frac{12399}{16987}$, B \$190.04.$0\frac{15520}{16987}$, C \$321.48.$3\frac{8279}{16987}$, D \$474.75.$1\frac{14763}{16987}$.

5. If two persons purchase a house jointly for \$4000, and afterwards let it for the yearly rent of \$650; what share of the yearly profit is each to receive, the one having contributed \$1850, and the other \$2150; and the taxes and other expenses being \$35.50 per year?

Ans. \$330.29.$3\frac{3}{4}$, and \$284.20.$6\frac{1}{4}$.

6. If the capital stock of a banker, amounting to \$500000, gain in 6 months \$22500, what is the gain per cent.,* and what does a stockholder receive, who holds 6 shares of \$50 each? Ans. \$4.50 per cent., and \$13.50.

7. Three merchants, A, B, and C, freight a ship from Madeira for Liverpool, 216 tuns of wine; of which A had 96, B 72, and C 48; the mariners, meeting with a storm at sea, were constrained, for the safety of their lives, to cast 45 tuns thereof overboard: how many of the 45 tuns has each merchant lost according to the rate of his adventure?

Ans. A 20, B 15, and C 10 tuns.

Compound Fellowship.

170. Rule.—Let all the times be of the same denomination, and multiply each stock by the time of its continuance in trade: then, using the products as stocks, proceed according to the rule for simple Fellowship.

Example.—A and B enter into partnership: A contributes \$600 for 13 months, and B \$800 for 10 months. Required the share of each in a gain of \$650.

Here, the products are \$7800 and \$8000, the sum of which is \$15800. Then, as \$15800 : \$650, or, by contraction, as \$316 : 13 : : \$7800 : \$320.89, A's share; and as \$316 : \$13 :: \$8000 :

$$
\begin{array}{r}
\$600 \times 13 = \$7800 \\
800 \times 10 = 8000 \\
\hline
\$15800 \\
\hline
\end{array}
$$

* It is proper to observe, that in banking concerns, the gain or loss per cent. is first calculated, and the dividend is declared at so much per cent.; so that *each person's share of the profit is found by multiplying his part of the capital by the gain per cent.*

$329.11, B's share. The sum of these is $650, which proves the correctness of the operation.*

Exercises.—1. A and B enter into partnership: A contributes $2800 for 5 months, and B $2660 for 6 months. Required the share of each in a gain of $330.

Ans. A $154.20.5+, B $175.79.4+.

2. A's stock $300 for 5, B's $500 for 8 months; whole gain $250. Ans. A $68.18.1$\frac{9}{11}$, B $181.81.8$\frac{2}{11}$.

3. A's stock $1200 for 18 months, B's $2500 for 24 months, and C's $3000 for 27 months: whole gain $2000.

Ans. A $265.68.2$\frac{178}{271}$, B $738.00.7$\frac{103}{271}$, C $996.30.9$\frac{241}{271}$.

4. Two merchants, A and B, entered into partnership, and on the 1st of May, 1826, each contributed $1000. On the 3d of June, A took out $200; on the 4th of July, B put in $350; on the 1st of September, A put in $500; and on the 16th of October, each took out $150. On the 1st of May, 1827, on making up accounts, it is found, that they have gained $1000. How is this gain to be divided between them? Ans. A $469.79.5$\frac{355}{831}$, B $530.20.4$\frac{476}{831}$.

5. Three merchants, A, B, and C, entered into partnership, and on the 1st of March each contributed £1000. On the 3d of May, A took out £300; on the 8th of June, B put in £360; and on the 20th of August, C withdrew £280. On the 1st of September, A put in £450; and on the 16th of October, each took out £180. On the 8th of January, of the following year, on making up accounts, it is found that they have gained £1250. How is this gain to be divided among them? Ans. A's share £341 1*s.* 11$\frac{3}{4}$*d.*, B's £512 12*s.* 1$\frac{1}{2}$*d.*, and C's £353 5*s.* 10$\frac{3}{4}$*d.*

Alligation.

171. Alligation is a rule which is chiefly employed in calculations respecting the compounding or combining of articles of different kinds.†

* *The reason of this rule* will be evident from the consideration, that a stock of $600 for 13 months would be the same as 13 times $600 for 1 month; and one of $800 for 10 months, the same as 10 times $800 for 1 month. Hence, if these increased stocks be employed, it is evident that since the times are then to be regarded as equal, the operation will proceed in the same manner as those in simple fellowship.

† This rule has its name from a Latin word which signifies to *bind*, because in the practical application of the rule, the quantities are usually *linked* or connected together by lines. It is a rule which is of little practical utility; being principally used in the solution of questions which are of rare occurrence in real transactions. Besides, every thing

PROBLEM.—*To find in what proportions quantities of given values must be taken, to form a compound of a given value.*

172. RULE.—Let the rates of the ingredients, all in the same denomination, be written in a line; and let the mean rate, in the same denomination, be written above them. Take two of the rates, one of which is greater and the other less than the mean rate, and write the difference between each of them and the mean rate, below the other. Proceed thus with the rates two by two, if there be more than two, till one or more differences stand below each. Then, if only one difference stand below any rate, it will be the quantity required at that rate; but if there be more than one, their sum will be the required quantity.*

Example 1. In what ratios must two kinds of brandy, worth 88 cents and \$1.12½ per gallon, respectively, be taken, to make a mixture worth 100 cents per gallon?

Here, the mean rate, 100 cents, is set above the other rates 88 cents, and 112½ cents. Then, the difference between 88 and the mean is set below 112½, and the difference between 112½ and the mean below 88.

	100	
88,		112½
12½		12

Hence, we find that the quantities must be in the ratio of 12½ to 12, or 25 to 24; that is, for every 25 gallons at 88 cents per gallon, 24 gallons at \$1·12½ per gallon, must be taken to form a compound worth 100 cents.†

that can be effected by this rule can be done in general in a better and easier way by Algebra. Hence, this article will be more circumscribed in its limits than it might otherwise have been. The following is the principal problem in this rule, and in fact the only one that belongs to it exclusively.

* The connecting or *linking* of the rates with crooked or curved lines, in the use of this rule, is attended with little advantage. Should that method be preferred, however, it can present no difficulty, as each rate less than the mean rate is to be connected with one greater, and each greater with one less, and the differences are to be set down below the rate to which the line directs.

† The correctness of this operation, and of the principle on which it depends, will appear manifest from the consideration, that in selling 12½ gallons at 100 cents, which cost only 88 cents, there is a gain of \$1·30; but in selling 12 gallons at 100 cents, instead of 112½ cents, there is a loss of \$1·30: and therefore, the gain on the one quantity balancing

Exam. 2. In what proportions must wines worth \$1·25, \$1·75, \$2·25, and \$2·50 per gallon, respectively, be mixed, so that the compound may be worth \$2 per gallon?

Here, by setting the difference between 125 and 200 below 225, and the difference between 225 and 200 below 125, and likewise by setting the difference between 175 and 200 below 175, and the difference between 250 and 200 below 175, we find, that for 25 gallons at \$1·25, we must take 75 gallons at \$2·25, 50 at \$1·75, and 25 at \$2·50. By using \$1·25 with \$2·50, and \$1·75 with \$2·25, a second answer is obtained, from which it appears, that if 50 gallons at \$1·25, 25 at \$1·75, 25 at \$2·25, and 75 at \$2·50, be mixed together, the compound will also be worth 2 dollars per gallon. It is scarcely necessary to observe, that any quantities in the same ratios, will serve the same purpose.*

200			
125,	175,	225,	250
25,	50,	75,	25
50,	25,	25,	75

The correctness of these results is proved by adding together the prices of 25 gallons at \$1·25, of 50 at \$1·75, of 75 at \$2·25, and of 25 at \$2·50. This will be found to be 35000 cents, which divided by $25+50+75+25=175$ gallons, gives exactly 200 cents for the mean rate; and thus the proof may be conducted in every case. This method of proof has been generally made a separate case of this rule,

the loss on the other, the value of the compound must be exactly the mean rate.

* With respect to the reason of the operation, it is obvious from what was said respecting the preceding example, that 25 gallons at \$1·25, and 75 at \$2·25 per gallon, would make a mixture worth \$2 per gallon; and likewise that a mixture of 50 at \$1·75, and 25 at \$2 50, would be worth the same per gallon; and it is evident, that both mixtures taken together must make a mixture of the same value also; and in the same way, every operation in this rule may be explained.

From these principles, it is also manifest, that if we multiply or divide 25 and 75 by any number, and 50 and 25 by any number, we should still have results that would satisfy the conditions of the question. Thus dividing the former by 25, and also the latter by 25, we find for answer 1 gallon at \$1·25, 3 at \$1·75, 2 at \$2·25, and 1 at \$2·50. Different answers may also be obtained by connecting the rates differently. Thus, by using 125 and 225 we get 25 and 75, and by connecting 125 and 250, we have 50 and 75, and then by using 175 and 225, we get 25 and 25. After this, by the requisite addition, we find 75, 25, 100, and 75, for the required quantities; and it is obvious, that by a still farther application of these principles, different answers may be found without limit.

200			
125,	175,	225,	250,
25	25	75	75
50		25	
75	25	100	75

and called with no great propriety *Alligation Medial.* The operation, according to the preceding rule, is usually *Alligation Alternate.*

Exam. 3. How much linen, at 2*s.* and at 2*s.* 5*d.* per yard, must be taken with 216 yards at 3*s.* 4*d.* that the whole may be worth 2*s.* 6*d.* per yard, at an average?

Here, by taking the differences, &c. as in the margin, we find that the quantities may be in the ratio of 10, 10, and 7. Then as 7 : 10 :: 216 : 308$\frac{4}{7}$. It appears, therefore, that 308$\frac{4}{7}$ yards, at 2*s.*, the same quantity at 2*s.* 5*d.*, and 216 yards at 3*s.* 4*d.*, will compose a parcel worth 2*s.* 6*d.* per yard, at an average: and various other answers might be found. This question belongs to what is usually called *Alligation Partial.*

	30	
24,	29,	40,
10,	10	6,
		1,
10,	10,	7,

Exam. 4. What quantities of tea, worth 96 cents, 90 cents, and 78 cents per lb., respectively, must be mixed together, to form a parcel containing 112 lbs., worth 88 cents per lb.?

Here, in finding the ratios, to make the first two terms different, the differences between 88 and 90, and 88 and 78, are set down twice. In this way it is found, that the quantities may be as 10, 20, and 12, or by halving each term, as 5, 10, and 6. Hence by the method of dividing into parts in a given ratio, (see page 212,) as 21, the sum of these, : 5 :: 112 lbs.; or by contracting, as 3 : 5 :: 16 lbs. to 26$\frac{2}{3}$ lbs.; and as 3 : 10 : : 16 lbs. : 53$\frac{1}{3}$ lbs.; and lastly, as 3 : 6 : : 16 lbs. to 32 lbs. It appears, therefore, that 26$\frac{2}{3}$ lbs. at 96 cents, 53$\frac{1}{3}$ lbs. at 90 cents, and 32 lbs. at 78 cents, will form a compound of 112 lbs. worth 88 cents per lb. This question belongs to what has generally been called *Alligation Total.* This name for this particular case of Alligation, as well as those already mentioned for the other cases, is properly falling into disuse.*

	80	
96,	90,	78,
10	10	8
	10	2
		2
10,	20,	12,

Exercises.—1. In what proportions must sugars, worth 13 cents, 11$\frac{1}{2}$ cents, and 9 cents per lb. respectively, be compounded, that the mixture may be worth 10$\frac{1}{2}$ cents per lb.? Ans. 3, 3, 7; 1, 2, 3, &c.

* As many questions in this rule admit of several answers, the pupil should prove his results in working the following exercises, particularly when his answers differ from those here given.

2. How much water must be added to a cask of spirits, containing 84 gallons, worth 94 cents per gallon, to reduce the value to $87\frac{1}{2}$ cents per gallon? Ans. $6\frac{42}{75}$ gal.

3. What quantities of three different kinds of raisins, worth 11 cents, 15 cents, and 22 cents per lb. respectively, must be mixed together, to fill a cask containing 200 lbs. and to be worth $16\frac{1}{2}$ cents per lb.?

Ans. $61\frac{1}{9}$ lbs., $61\frac{1}{9}$ lbs., $77\frac{7}{9}$ lbs.

4. How much land, worth $35 per acre, must be added to a farm containing 51 acres, 2 roods, 20 poles, worth $69 per acre, to reduce the average value of both together to $45·50? Ans. 115 *acres*. 2*r*. $6\frac{2}{3}$ *poles*.

5. A box of linen, containing 1200 yards, worth at an average 36 cents per yard, consists of two kinds, one worth $32\frac{1}{2}$ cents, and the other worth $45\frac{1}{2}$ cents per yard. How much of each kind does it contain?

Ans. $876\frac{12}{13}$ *yds*. and $323\frac{1}{13}$ *yds*.

6. How much wine at $1·75 per gallon, must be added to a mixture, consisting of 41 gallons at $1·14, and 59 gallons at $1·28, to make the compound worth $1·38 per gallon? Ans. $52\frac{7}{15}$ galls.

7. In a factory the persons employed consist of boys at 32 cents, and girls at 22 cents per day; and the amount of the wages of the whole is the same as if each of them had 25 cents. Required the number of boys, the number of girls being 21. Ans. 9 boys.

Questions.

Repeat the rule for dividing a given quantity into parts which shall have to each other given ratios.

What is Fellowship, and how is it distinguished?

What is Simple Fellowship?

What is Compound Fellowship?

Repeat the rule for Simple Fellowship and Bankruptcy.

Repeat the rule for Compound Fellowship.

What is the use of Alligation?

Repeat the rule for finding in what proportions, quantities of given values must be taken to form a compound of a given value.

CHAPTER XI.

INVOLUTION AND EVOLUTION.

173. A *power* of any number is the product obtained by the continual multiplication of that number, repeated a certain number of times as factor.

174. A number, in relation to any power of it, is called the *root* of that power. When the proposed number is used *twice* as factor, the product is called the *second power*, or usually the *square*, of that number; when *three times*, the *third power* or *cube*; when *four times*, the *fourth power*; when *five times*, the *fifth power*, &c.

Powers are often denoted by writing at the upper and right side of the proposed number, the number, in small figures, which shows how often the proposed number is repeated as factor. This number is called the *index*, or *exponent*, of the power.

Thus, $5\times5=25$, is the second power, or the square of 5, and may be written 5^2, where 2 is the index; $7\times7\times7\times7=2401$, is the fourth power of 7, and may be written 7^4, where 4 is the index or exponent, &c.

175. The method of finding any assigned power of a given number, or, as it is also expressed, the method of *raising* a number to any proposed power, is called *Involution*.

INVOLUTION.

PROBLEM.—*To find any assigned power of a given number; or to raise a given number to any proposed power.*

176. RULE.—Find the continual product of the given number repeated as factor, as often as there are units in the index of the proposed power.

The process may often be abbreviated by multiplying together powers already found. In this case, the index of the power thus found, is equal to the *sum* of the indices of the powers multiplied together.

Example 1. Required the fifth power of 23.

Here, by multiplying 23 by itself, we find 529 for the second power of 23. By multiplying this by 23, we find 12167 for the third power. By proceeding in like manner, we find the fourth power to be 279841, and the fifth power to be 6436343.

Multiply	23=1st	power
	23	
Multiply	529=2d	do.
	23	
Multiply	12167=3d	do.
	23	
Multiply	279841=4th	do.
	23	
Answer	6436343=5th	do.

The same result might also have been found by multiplying the second power 529, by itself, and the product 279841, which is the fourth power by 23. The same result would also be obtained by multiplying the third power by the second.*

Exam. 2. Required the fifth power of $\frac{3}{8}$.

The fifth power of 3 is 243, and the fifth power of 8 is 32768: the answer therefore is $\frac{243}{32768}$. The *reason* of this is evident from multiplication of fractions.

Exam. 3. What is the third power of $1\frac{1}{4}$?

This, by reduction to an improper fraction, becomes $\frac{5}{4}$; and by involving the numerator and denominator each to the third power, we find for answer $\frac{125}{64}$, or $1\frac{61}{64}$.†

Exam. 4. Required the sixth power of 1·12.

By raising this to the third power, in the way already shown, we find 1.404928, and this being multiplied by itself, as in the margin, we find for the sixth power 1.973822685184.

```
      1.404928= 3d power
      1.404928
      --------
      11239424
      2809856
     12644352
     5619712
    56197120
    14049280
   -------------
   1.973822685184
```

* In like manner, if it were required to find the twelfth power of a number; multiply the number by itself to find the second power; the second power by itself to find the fourth power; the fourth power by itself to find the eighth power; and, lastly, the eighth power by the fourth to find the twelfth.

† Each of the last two examples might have been wrought by reducing the fractions to decimals, and then working by the general rule.

Exercises.—1. Required the square of 678.
Ans. 459684.

2. Required the cube or third power of 119.
Ans. 1685159.

3. Required the fourth power of 75. Ans. 31640625.

4. Required the fourth power of 4.367.
Ans. 363.691179.

5. What is the sixth power of $\frac{2}{3}$? Ans. $\frac{64}{729}$.

6. What is the fifth power of $2\frac{3}{4}$? Ans. $157\frac{283}{1024}$.

7. What is the fourth power of $3\frac{2}{7}$? Ans. $116\frac{1147}{2401}$.

8. What is the fifth power of 0.29? Ans. .0020511149.

9. What is the sixth power of 6.03?
Ans. 48073.293078275529.

10. How many acres does a square contain, whose sides are each 290 poles? Ans. 525*acrs.* 2*r.* 20*p.*

11. How many cubic feet does a cube contain, whose sides are each 6 feet? Ans. 216 cubic feet.

EVOLUTION.

177. *Evolution* is the method of finding an assigned root of a given number; or, which amounts to the same thing, *Evolution*, or *the extraction of roots*, being just the reverse of involution, is the method of finding a number which raised to a proposed power, will produce a given number.

178. The *index* of a root is a fraction whose denominator denotes the order of the root, and whose numerator is a unit. The root of a number is also expressed by prefixing to the number the sign $\sqrt{}$ with the number above it, which denotes the order of the root. In case of the square, or second root, however, the number 2 is omitted.

Thus, the fourth root of 10 is denoted by $10^{\frac{1}{4}}$, or $\sqrt[4]{10}$, and means a number whose fourth power is 10; and the second or square root of 7 is written $7^{\frac{1}{2}}$ or $\sqrt{7}$, and means a number, such, that if it be multiplied by itself, the product will be 7.*

* Any power of a given number may be found exactly; but we cannot, conversely, find every root of a given number exactly. Thus, we know the *square root* of 4 exactly, being 2; but we cannot assign exactly

To facilitate the extraction of the square and cube roots, it may be proper for the pupil to be familiar with the following tables:

Table I.—The square of $1=1$; of $2=4$; of $3=9$; of $4=16$; of $5=25$; of $6=36$; of $7=49$; of $8=64$; of $9=81$.

Table II.—The cube of $1=1$; of $2=8$; of $3=27$; of $4=64$; of $5=125$; of $6=216$; of $7=343$; of $8=512$; of $9=729$.

PROBLEM I.—*To extract the second, or square root of a given number.*

179. RULE.—Commencing at the unit figure, cut off periods of two figures each, till all the figures of the given number are exhausted.* The first figure of the required root will be the square root of the first period, or of the greatest square contained in it, if it be not a square itself. Subtract the square of this figure from the first period; to the remainder annex the next period for a dividend; and, for part of a divisor, double the part of the root already obtained. Try how often this part of the divisor is contained in the dividend wanting the last figure, and annex the figures thus found to the parts of the root of the divisor already determined. Then multiply and subtract as in division; to the remainder bring down the next period; and, adding to the divisor the figure of the root last found, proceed as before.

the *cube root* of 4. So again, though we know the cube root of 8, being 2, we cannot assign exactly the *square root* of 8. But of 64 we can assign both the square root and the cube root, the former being 8, the latter 4. By means of decimals, we can, in all cases, *approximate* the root, to any proposed degree of exactness. Those roots which only approximate, or which is the same, those roots whose values cannot be accurately expressed in numbers, are called *irrational* numbers, *surds*, or *surd roots*, as $\sqrt{2}$, $\sqrt[3]{5}$, $\sqrt[4]{9}$, &c. while those which can be found exactly are called rational; as $\sqrt{9}=3$, $\sqrt[3]{125}=5$, $\sqrt[4]{16}=2$, &c.

* In dividing a decimal, or a number consisting of a whole number with a decimal, into periods, the division must also commence at the unit figure or decimal point, and must be continued both ways, if there be a whole number; and if there be an odd figure at the end of the decimal, a cipher, or if it be a periodical decimal, the figure that would next arise from its continuation, must be annexed; thus, 417.245 will be divided thus 4′17′.24′50: 41.66666, &c. thus, 41′.66′66′66: and .567 thus, .56′70, &c.

If any thing remain, after continuing the process till all the figures in the given number have been used, proceed in the same manner to find decimals, adding, to find each figure, two ciphers, or if the given number end in an interminate decimal, the two figures that would next arise from its continuation. *To extract the root of a fraction*, reduce it to its simplest form, if it be not so already, and extract the root of both terms, if they be complete powers: otherwise divide the root of their product by the denominator.

The root may also be found by reducing the fraction to a decimal, if it be not such already, and taking the root of the decimal.

Example 1. Required the square root of 365.

```
 3'65(19.1049
 1
 ----
 29)265
  9 261
 -------
 381) 400
   1  381
 ------------
 38204)190000
     4 152816
 ------------
 382089)3718400
      9 3438801
 ---------------
 382098  )279599
```

Here, by placing a separating mark between 3 and 6, the given number is divided into the two periods, 3 and 65. Then, 1, the greatest integral square contained in 3, is put down in the quotient, and its square taken from the first period. To the remainder the next period is brought down, which makes for dividend 265. The first part of the divisor is found by doubling 1, the first part of the quotient. In finding, in the next place, what figure must be annexed to the part of the root already found, though 2 would be contained 13 times in 26, yet we try 9, as we know the next figure cannot be greater than 9. We annex 9, therefore, to the parts of the root and of the divisor already found, and multiplying 29 by 9, we find for product 261, and for remainder 4. Hence we have for root 19, and for remainder 4. Now to find decimal figures, a point is put in the quotient, two ciphers annexed to the remainder, and 9, the figure last found, added to 29, the former divisor, we have then for dividend 4.00, for part of a divisor 38. This part of the divisor is contained once in 40; and therefore the first figure of the decimal is 1, which is also annexed to 38. In working for the next figure, we have the divisor 382, which not being contained in 190, a cipher is annexed, to the parts of the root and divisor already found, and two ciphers annexed to the dividend to

find another figure. The rest of the work proceeds in the same manner, and the root true to four places of decimals, is found to be 19.1049. The truth of this result is proved by multiplying 19.1049 by itself, and adding the remainder to the product; as the result will be exactly 365.*

2. Required the square root of $\frac{5}{7}$.

Here, the square root of 35 ($=5\times7$), which is found to be 5.9160798, divided by 7, the result will be .84515425, the root required. The same result would be obtained by extracting the root of .71′42′85′71, the decimal equivalent to the given fraction. The reason of this is evident, since $\frac{5}{7}=\frac{5\times7}{7\times7}=\frac{35}{49}$, and the square root of $\frac{35}{49}$ is equivalent to the square root of 35 divided by the square root of 49, or 7.

3. Required the square root of $2\frac{1}{4}$.

Here, $2\frac{1}{4}$ is equivalent to $\frac{9}{4}$, and the square root of $\frac{9}{4}$ is equal to $\frac{3}{2}$, or $1\frac{1}{2}$. This result might also be obtained by extracting the root of 2.25. This would be found to be 1.5, or $1\frac{1}{2}$, as before.

Exercises.—1. Required the square root of 106929. Ans. 327.

2. Required the square root of 152399025. Ans. 12345.

3. Required the square root of 5499025. Ans. 2345.

4. Required the square root of 78.5. Ans. 8.8600225.

5. Required the square root of .006. Ans. .02449.

6. Required the square root of $11\frac{1}{9}$. Ans. 3.3′, or $3\frac{1}{3}$.

7. Required the square root of $1\frac{13}{36}$. Ans. 1.16′, or $1\frac{1}{6}$.

* *The principle* on which the preceding rule depends, is, *that the square of the sum of two numbers is equal to the squares of the numbers with twice their product.* Thus, the square of 34 is equal to the squares of 30 and of 4 with twice the product of 30 and 4; that is, to $900+2\times30\times4+16=1156$. Here, in extracting the second root of 1156, we separate it into two parts, 1100 and 56. Thus, 1100 contains 900, the square of 30, with the remainder 200; the first part of the root is therefore 30, and the remainder 200+56, or 256. Now, according to the principle above mentioned, this remainder must be twice the product of 30 and the part of the root still to be found, together with the square of that part. Now, dividing 256 by 60, the double of 30, we find for quotient 4; then this part being added to 60, the sum is 64, which being multiplied by 4, the product 256, is evidently twice the product of 30 and 4, together with the square of 4. In the same manner the operation may be illustrated in every case. The rule, however, is best demonstrated by Algebra; see my treatise on the subject, page 231, second edition.

Problem II.—*To extract the third or cube root of a given number.*

180. Rule.—Commencing at the unit figure, cut off periods of three figures each, till all the figures of the given number are exhausted. Then find the greatest cube number contained in the *first* period, and place the cube root of it in the quotient. Subtract its cube from the first period, and bring down the next three figures; divide the number thus brought down by 300 times the square of the first figure of the root, and it will give the second figure; add 300 times the square of the first figure, 30 times the product of the first and second figures, and the square of the second figure together, for a divisor; then multiply this divisor by the second figure, and subtract the result from the dividend, and then bring down the next period, and so proceed till all the periods are brought down.

To extract the cube root of a fraction, reduce it to a decimal, and then extract the root; or multiply the numerator by the square of the denominator, find the cube root of the product, and divide it by the denominator.

The cube root of a mixed number is generally best found by reducing the fractional part to a decimal, if it be not so already, and then extracting the root. It may also be found by reducing the given number to an improper fraction, and then working according to the preceding directions.

Example.—Required the cube root of 48228544.

48'228'544(364	
27	
3276)21228	Divided by $300\times3^2=2700$
19656	$30\times3\times6=\ 540$
	$6\times6=\ \ 36$
393136)1572544	
1572544	1st divisor 3276

$$
\begin{array}{rr}
\text{Divide by } 36^2 \times 300 = & 388800 \\
30 \times 36 \times 4 = & 4320 \\
4 \times 4 = & 16 \\
\hline
\text{2d divisor} & 393136 \\
\hline
\end{array}
$$

Here, by dividing the given number into periods, we find for the *first* period 48 : then 3, the root of the greatest integral cube contained in 48, is put in the quotient, and its cube taken from the first period. To the remainder the next period is brought down, which makes for dividend 21228. The first part of the divisor is found by multiplying the square of 3 by 300 ; in finding, in the next place, what figure must be annexed to the part of the root already found, though 2700 would be contained 8 times in 21228, yet we can readily find by trial, that 6 is the figure which must be put in the quotient, since 2700 is not the complete divisor. Now, to find the complete divisor, we must add 30 times the product of 3 and 6, or 540, and also the square of 6, or 36, to 2700 ; then, multiplying the sum thus found 3276 by 6, and subtracting the product from 21228, there remains 1572. To this remainder the next period 544 is brought down. The rest of the work proceeds in the same manner, and the root is found to be 364. The truth of this is proved by raising 364 to the third power, as the result will be exactly 48228544.*

* The reason of the preceding rule will appear evident from the following illustration. The cube of 25, for instance, is equivalent to the cube of 20 added to the cube of 5, together with the sum of $300 \times 4 \times 5 + 30 \times 2 \times 5 \times 5$; or, which is the same thing, 25 is equal to $20+5$, and, therefore, 25 cubed is equal to $20+5$ cubed ; but $20+5$ cubed is equivalent to $8000+300 \times 4 \times 5+30 \times 2 \times 5 \times 5+125$, or to $20^3+(300 \times 4+30 \times 2 \times 5+5 \times 5) \times 5=48228544$.

$$
\begin{array}{lll}
\text{Thus,} & \left.\begin{array}{l} 20+5 \\ 20+5 \end{array}\right\} & \text{Multiplied} \\
\cline{2-2}
 & \begin{array}{l} 20 \times 20+5 \times 20 \\ \qquad\quad +5 \times 20+25 \end{array} & \\
\cline{2-2}
\text{Multiplied} & \left\{\begin{array}{l} 20 \times 20+2 \times 5 \times 20+25=\text{2d power.} \\ 20+5 \end{array}\right. & \\
\cline{2-2}
 & \begin{array}{l} 20 \times 20 \times 20+2 \times 5 \times 20 \times 20+20 \times 25 \\ \qquad\qquad\qquad 5 \times 20 \times 20+2 \times 20 \times 25+125 \end{array} & \\
\cline{2-2}
 & 8000+3 \times 5 \times 20 \times 20+3 \times 20 \times 25+125=\text{3d power.} & \\
 & \text{or } 8000+300 \times 4 \times 5+30 \times 2 \times 25+125. &
\end{array}
$$

Exercises.—1. Required the cube root of 34567.
Ans. 32.57521043, &c.

2. Required the cube root of 782140.
Ans. 92.1357479, &c.

3. Required the cube root of 123456789.
Ans. 497.933859, &c.

4. Required the cube root of 389017. Ans. 73.

5. Required the cube root of 1092727. Ans. 103.

6. Required the cube root of 146708.483.
Ans. 52.74, &c.

7. What is the cube root of .0001357? Ans. .05138, &c.

8. What is the cube root of $13\frac{2}{3}$? Ans. 2.3908.

9. What is the cube root of $\frac{1520}{5130}$? Ans. $\frac{2}{3}$.

10. What is the cube root of $\frac{4}{15}$? Ans. .6436595897, &c.

Questions.

What is a power?
What is called the root of a number?
How are powers denoted?
What is involution?
Repeat the rule for finding any power of a given number.
What is evolution?
How is the root of a given number denoted?
Repeat the rule for extracting the square root of a given number.
Repeat the rule for extracting the cube root of a given number.

CHAPTER XII.

POSITION.*

181. Position is a rule by which, from the assumption of one or more false answers to a problem, the true one is obtained.

Hence, the rule is evident. In the same manner, the operation may be illustrated in every case. For a demonstration of this rule in general terms, see my Treatise on Algebra, Theoretical and Practical.

* This rule is sometimes called *the rule of false*, or *the rule of false position*, or *the rule of trial and error*. It might properly be called *the rule of supposition*.

It admits of two varieties, *single position* and *double position*.

In *single position* the answer is obtained by *one* assumption: in *double position* it is obtained by *two*.*

Single Position.†

182. Rule.—Assume any number, and perform on it the operations mentioned in the question as being performed on the required number. Then, as the result thus obtained is to the assumed number, so is the result given in the question to the number required.

Example.—Required a number to which if one-half, one-third, one-fourth, and one-fifth of itself be added, the sum may be 1664.

Suppose the number to be 60: then, if to 60 one-half, one-third, one-fourth, and one fifth of itself be added, the sum is 137. Hence, according to the rule, as 137 : 60 :: 1644 : 720, the number required. The truth of the result is proved by adding to 720 one-half, one-third, &c. of itself, and the sum is found to be 1644.‡

Exercises.—1. Divide \$2000 between A, B, and C, giving A as much as B, and a fifth part more, and C as much as both together.

Ans. A's part \$545.45.4$\frac{6}{11}$, B's \$454.54.5$\frac{5}{11}$, C's \$1000.

2. One-third of a ship belongs to A, and one-fifth to B, and A's part is worth \$1000 more than B's: required the value of the ship. Ans. \$7500.

3. A father bequeaths to his three sons \$7000, in such a manner, that if the share of the eldest be multiplied by 5,

* Single position may be employed in resolving problems in which the required number is any how increased or diminished in a given ratio; such as when it is increased or diminished by any part of itself, or when it is multiplied or divided by any number.

† Every question that can be resolved by this rule, may also be resolved by the rule for double position, or without position, by some of the preceding rules; and hence this rule is of little importance.

‡ The number 60 was here assumed, not as being near the truth, but as being a multiple of 2, 3, 4, and 5; and in this way the operation was kept free from fractions. By the assumption of any other number, however, the answer would have been found correctly, but often not so easily. The reason of the operation is obvious from the principles of proportion.

that of the second by 6, and that of the third by 7; the products are all equal. What are their shares?

Ans. \$$1962\frac{66}{107}$, \$$2289\frac{77}{107}$, \$$2747\frac{71}{107}$.

4. The number of a gentleman's horses is two-fifths of the number of his cows and oxen, and for every 4 of the cows and oxen he has 11 sheep. Required the number of each, the number of the sheep exceeding that of the horses by 141. Ans. 24 horses, 60 cows and oxen, and 165 sheep.

Double Position.

183. RULE I.—Assume two different numbers, and perform on them separately the operations indicated in the question: then, as the difference of the results thus obtained is to the difference of the assumed numbers, so is the difference between the true result and either of the others to the correction to be applied, by addition or subtraction, as the case may require, to the assumed number which gave this result.*

Example 1. Required a number from which if 2 be subtracted, one-third of the remainder will be 5 less than half the required number.

Here, suppose the required number to be 8, from which take 2, and one-third of the remainder is 2. This being taken from one-half of 8, the remainder is 2, the *first result*. Suppose again the number to be 32, and from it take 2; one-third of the remainder is 10, which being taken from the half of 32, the remainder is 6, the *second result*. Then, the difference of the results being 4, the difference of the assumed numbers 24, and the difference between 5, the true result, and 6, the result nearer it, being 1; as 4 : 24 : : 1 : 6, the correction to be subtracted from 32, since the result 6 was too great. Hence, the required number is 26.

* This rule, which was first published in substance by *Bonnycastle, in his Arithmetic*, is, according to *Professor Thomson*, the simplest and easiest that has yet appeared for the resolution of questions in which the given result is *a known number, independent on the required number*. See *Professor Thomson's* observations on this rule in his Treatise on Arithmetic.

184. Rule II.—Having assumed two different numbers, perform on them separately the operations indicated in the question, and find the *errors* of the results. Then, as the difference of the errors, if both results be too great or both too little, or as the sum of the errors, if one result be too great and the other too small, is to the difference of the assumed numbers, so is either error to the correction to be applied to the number that produced that correction.

2. If one person's age be now only four times as great as another's, though 7 years ago it was 6 times as great, what is the age of each?

Here, suppose the age of the younger to be 12 years; then would the age of the older be 48. Take 7 from each of these, and there will remain 5 and 41, their ages 7 years ago. Now, 6 times 5 are 30, which, taken from 41, leaves an error of 11 years. By supposing the age of the younger to be 15, and proceeding in a similar manner, the error is found to be 5 years. Hence, as 6, the difference of the errors, (both results being too small,) is to 3, the difference of the assumed numbers, so is 5, the less error to $2\frac{1}{2}$, the correction; which being added to 15, the sum, $17\frac{1}{2}$, is the age of the younger, and consequently that of the older must be 70.*

3. Required a number, to which if twice its square be added, the sum will be 100. Ans. 6.8254.

* Both the rules above given depend on the principle, that the differences between the true and the assumed numbers are proportional to the differences between the result given in the question and the results arising from the assumed numbers. This principle is quite correct in relation to all questions which in Algebra would be resolved by what is usually called Simple Equations, but not in relation to any others; and hence, when applied to others, it gives only approximations to the true results. In this case, the assumed numbers should be taken as near the true answer as possible. Then, to approximate the required number still more nearly, assume for a second operation the number found by the first, and that one of the first two assumptions which was nearer the true answer, or any other number that may appear nearer it still. In this way, by repeating the operation as often as may be necessary, the true result may be approximated to any assigned degree of accuracy. When applied in this way, Double Position is of considerable use in Algebra, affording in complicated expressions a very convenient mode of approximating the roots of equations, and of finding the values of unknown quantities.

It is easy to see that this number must be between 6 and 7. These numbers being assumed, therefore, the sum of 6 and twice its square is 78, and the sum of 7 and twice its square 105. Then, as 105—78 : 7—6 : : 105—100 : .18; which being taken from 7, the remainder, 6.82, is the required number nearly. To this let twice its square be added, and the result is 99.8448. Then, as 105—99.8448 : 7—6.82 : : 105—100 : .1746; which being taken from 7, the remainder is 6.8254, the required number still more nearly; and if the operation were repeated, with this and the former approximate answer, the required number would be found true for seven or eight figures.

Exercises.—1. Required a number, from which if 84 be taken, 3 times the remainder will exceed the required number by one-fourth of itself. Ans. 144.

2. One being asked how old he was, answered, that the product of $\frac{1}{25}$th of the years he had lived, being multiplied by $\frac{5}{6}$ths of the same, would be his age. What was his age? Ans. 30 years.

3. After A had lent $10 to B, he wanted $8 in order to have as much money as B; and together they had $60. What money had each at first? Ans. A $36, and B $24.

4. Two persons began to play with equal sums of money; the first lost $14, the other won $24, and then the second had twice as many dollars as the first. What sum had each at first? Ans. $52.

5. A farmer had two flocks of sheep, each containing the same number; from one of these he sells 39, and from the other 93; and finds twice as many remaining in one as in the other. How many did each flock originally contain? Ans. 147.

6. A sum of money was divided between two persons, A and B, so that the share of A was to that of B as 5 to 3; and exceeded five-ninths of the whole sum by 50 dollars. What was the share of each person? Ans. A's $450, and B's $270.

7. Required a number which exceeds 3 times its square root by 10. Ans. 25.

8. Required a number, to which if twice its square and 3 times its cube be added, the sum will be 2000. Ans. 8.506744.

Questions.

What is position?
How many kinds of position are there?
What is single position?
What is double position?
Repeat the rule for working questions in single position.
Repeat the first rule for working questions in double position.
Repeat the second rule.

CHAPTER XIII.

EXCHANGE.

185. *Exchange* is the method of finding what sum of the money of one country is equivalent to any given sum of the money of another.

186. By the *par of exchange* between two countries is meant the intrinsic value of the money of one compared with that of the other, and estimated by the weight and fineness of the coins.*

187. The *course of exchange*, at any particular time, is the sum of the money of one country which at th time is given for a fixed sum of the money of ano country. This is seldom at par, but is continua rying according to the circumstances of trade.†

There are two kinds of money; *real* an *imaginary*. All gold, silver, and copper

* The par continues the same, so long as are of the same weight, fineness, and nom ception to this, when the standard coins case, the par varies as the comparative silver is the standard coin, in gen gold is the standard coin in Grea the continent of Europe, silver

† The variation in the co cumstances, which, as wel exchanges, do not accor Such discussions belo *Arithmetic*

real money: and the *imaginary money* is a denomination used to express money of which there is no real species current, precisely of the same value, as a *livre* in France; and a *pound* and a *penny* in the United States. In some countries they keep their accounts and calculate their payments in imaginary money.

188. Exchange in this country may be distinguished into *Domestic* and *Foreign*.

Domestic exchange consists in the reduction of the currencies of either state into that of any other. Accounts are generally kept at present in the United States in dollars and cents; hence, domestic exchange is falling into disuse, or if it be used at all, it is for the purpose of changing the nominal currencies of the States to dollars, or of reducing dollars to the nominal currencies of the States.

All the calculations in exchange may be performed by the Rule of Proportion. In foreign exchanges, one place always gives another a *fixed sum or piece of money* for a *variable one.* The former is called the *certain price* or *rate;* the latter the *variable price* or *rate.**

DOMESTIC EXCHANGE.

PROBLEM I.—*To reduce the currencies of the United States to dollars and cents.*

189. RULE.—As the value of a dollar in the given currency is to the sum whose value is to be found in dollars and cents, so is one dollar to the dollars and cents required.

The coins of the United States are gold, silver, and copper. The gold coins are the eagle, which weighs 270 grains, half eagle, and quarter eagle. The standard gold for coinage, consists of 11 parts of pure gold, and 1 of alloy; which is usually a mixture of silver and copper.

* Thus, in exchanges with Amsterdam, London receives for one pound sterling a number of skillings and pence of the money of Amsterdam, *which is continually varying*, being at one time perhaps 36 skillings and 8 pence, at another 37 skillings and 7 pence, &c. Here one pound is the certain price, or rate, and 36 skillings and 8 pence, &c. *variable price*, or rate. In exchanges with Portugal, on the contrary, London gives for 1 milrea some times 65 pence, some times 68 pence, &c. In this case, 1 milrea is the *fixed price*, and 65 pence, or 68 pence, &c. the *variable price.*

The silver coins are the dollar, which weighs 17 dwts. 8 grains, $\frac{1}{2}$ dollar, $\frac{1}{4}$ dollar, $\frac{1}{8}$ dollar, $\frac{1}{16}$ dollar; the standard silver consists of 15 dwts. $11\frac{1}{4}$ grains of pure silver, and 1 dwt. $20\frac{3}{4}$ grains of alloy, which is usually copper.

The copper coins are cent, which weighs 208 grains, and a half cent.

190. In the New-England States, Virginia, and Kentucky, the nominal value of a dollar is 6 shillings; New-York and North Carolina 8*s.*; Pennsylvania, New-Jersey, Delaware, and Maryland 7*s.* 6*d.*; South Carolina and Georgia, 4*s.* 6*d.**

Example 1. How many dollars and cents are equivalent to £756 8*s.* New-York currency?

Here, 8*s.* is the value of a dollar, New-York currency, and £756 8*s.* is the given sum; hence the analogy is evident.

```
   s.    £   s.      $      $
As 8 : 756   8 : : 1 : 1891.
        20
      ------
    8)15128
      ------
     $1891 Answer.
```

In this example, the operation may be performed by multiplying the pounds by 5, and dividing the product by 2; then, adding the value of the shillings and pence, if there be

* It is proper to remark, that in the United States, 12 pence make 1 shilling, 20 shillings make 1 pound; but the pound, shilling, and pence in the different states are not the same in value; for $12\frac{1}{2}$ cents, or 12 pence New-York currency, is equivalent to 9 pence New-England currency, =11 pence, Pennsylvania, &c. The following Table for reducing pence, New-York money, to cents, may be useful to the student; and a similar Table may be readily calculated for any other of the United States.

TABLE.

s.	*d.*	*cts.*	*s.*	*d.*	*cts.*	*s.*	*cts.*
0	6	= $6\frac{1}{4}$	6	0	= 75	12	=150
1	0	= $12\frac{1}{2}$	6	6	= $81\frac{1}{4}$	13	= $162\frac{1}{2}$
1	6	= $18\frac{3}{4}$	7	0	= $87\frac{1}{2}$	14	=175
2	0	=25	7	6	= $93\frac{3}{4}$	15	= $187\frac{1}{2}$
2	6	= $31\frac{1}{4}$	8	0	=100	16	=200
3	0	= $37\frac{1}{2}$	8	6	= $106\frac{1}{4}$	17	= $212\frac{1}{2}$
3	6	= $43\frac{3}{4}$	9	0	= $112\frac{1}{2}$	18	=225
4	0	=50	9	6	= $118\frac{3}{4}$	19	= $237\frac{1}{2}$
4	6	= $66\frac{1}{4}$	10	0	=125	20	=250
5	0	= $62\frac{1}{2}$	10	6	= $131\frac{1}{4}$		
5	6	= $68\frac{3}{4}$	11	0	= $137\frac{1}{2}$		

any, to the result, which is readily found from the table at the bottom of the last page.*

	£	s.			£	s.
Thus,	756	8		Or thus,	756	8
		5				5
	2)3780				2)3782	0
	$1890				$1891 Ans.	
8s=	1					
	1891					

Exercises.—1. Reduce £176 19s. 6d. North Carolina currency to dollars and cents. Ans. $442.43.7$\frac{22}{48}$.

2. How many dollars and cents are equivalent to £870 10s. New-England currency? Ans. $2901.66.6$\frac{2}{3}$.

3. How many dollars and cents are equivalent to £800, Pennsylvania money? Ans. $2133.33.3$\frac{1}{3}$.

4. How many dollars and cents are equivalent to £120 4s. South Carolina and Georgia money? Ans. $515.14.2$\frac{6}{7}$.

5. How many dollars and cents are equivalent to £175 16s. 8d. Kentucky money? Ans. $586.11.1$\frac{1}{9}$.

6. How many cents are equivalent to 1600 pence New-York currency? Ans. 1666$\frac{2}{3}$ *cts.*

7. How many cents are equivalent to 16s. 8d. Virginia money? Ans. 277$\frac{7}{9}$ *cts.*

8. How many cents are equivalent to 15s. 6d. Georgia money? Ans. $3.32.1$\frac{3}{7}$.

* The reason of this contraction is evident, since 8s., the value of a dollar, New-York currency, is the $\frac{2}{5}$ of 20s. or 1 pound; that is $\frac{8}{20}=\frac{2}{5}$ *l.*=$1. In like manner, we may abbreviate the method of reducing the money of the other states to dollars and cents. Thus, multiply the pounds New-England, Virginia, and Kentucky currencies by 10, and divide the product by 3, and the result will be dollars, because $\frac{3}{10}$ of a pound is equal to one dollar. Again, New-Jersey, Pennsylvania, Delaware, and Maryland currencies, is reduced to dollars, by multiplying the pounds by 8, and dividing the result by 3; since 7s. 6d. the value of a dollar in these states, is the $\frac{3}{8}$ of a pound, or *l.* $\frac{3}{8}$=$1. South Carolina and Georgia currencies are reduced to dollars, by multiplying the pounds by 30, and dividing the product by 7; because 4s. 8d. the value of a dollar in those states, is equivalent to $\frac{7}{30}$ *l.* or $\frac{56}{240}$ *l.*=$\frac{7}{30}$ of a dollar. The *converse* of this is also true; that is, dollars may be reduced to the currency of any state by multiplying the dollars by the numerator of the ratio of the value of the dollar in that state to the pound, and then dividing the result by the denominator.

PROBLEM II.—*To reduce dollars and cents to the currencies of the United States.*

191. RULE.—As one dollar is to the given sum, so is the value of a dollar in the given currency to the value of the given sum in the same currency.

Example. How much money, South Carolina currency, is equivalent to \$1200?

\$ \$ s. d.			Or thus.
As 1 : 1200 :: 4 8 :			\$1200
56 12			7
7200 56			30)840'0
6000			£280 Ans.
12)67200			
20)560'0			
£280 Ans.			

Exercises.—1. How much money of New-York currency is equivalent to \$1300·50? Ans. £520 4s.

2. How much money of Kentucky, is equivalent to \$2000? Ans. £600.

3. How much money of New-Jersey, is equivalent to \$1827? Ans. £685 2s. 6d.

PROBLEM III.—*To change the currency of any State into that of any other.*

192. RULE.—As the value of a dollar in the given currency is to the value of a dollar in the currency required, so is the given sum to the sum required.

Example. What is the value of £376 10s. New-York currency, in Virginia?

s. s. £ s.	Or thus.
As 8 : 6 :: 376 10 :	£ s.
20	376 10
4 3	3
7530	4)1129 10
3	£282 7s. 6d.
22590	

4)22590

2'0)564'7 6

£282 7s. 6d.

Or thus.

	£	s.	d.
	376	10	
Subtract	94	2	6
	£282	7	6*

Exercises.—1. What is the value of £376 10s., Rhode Island currency, in Charleston, South Carolina? Ans. £292 16s. 8d.

2. What is the value of £870 10s. 6d., New-Jersey currency, in Virginia? Ans. £696 8s. 4$\frac{4}{5}$d.

3. What is the value of £3125, Georgia money, in New-York? Ans. £5357 2s. 10$\frac{1}{4}$+$\frac{1}{7}$d.

4. What is the value of £3700, Boston money, in New-York? Ans. £4933 6s. 8d.

5. What is the value of £1760 10s., Baltimore currency, Maryland, in Milledgeville, Georgia? Ans. £1095 8s. 5$\frac{1}{3}$d.

FOREIGN EXCHANGE.

193. Rule.—Place as the second term in the analogy, that sum whose value is to be found in the money of another country: make that term of the rate which is of the same kind with it, the first term of the analogy; and the remaining term of the rate the third term; then work the analogy in the usual way.†

* Here, it is evident, that 1*l*. Virginia money is equal to 1$\frac{1}{4}$*l*. New-York money, since 6s. Virginia money is equivalent to 8s. New-York money, both being equivalent to $1; therefore, 1*l*. Virginia money is equivalent to 1$\frac{1}{4}$*l*. New-York, as 8—$\frac{8}{4}$=6. Hence, also, by adding $\frac{1}{3}$ of the given sum, New-England or Virginia currency, to the same sum, the result may be considered as New-York currency. It may be shown, in like manner, how the method of changing the currency of any one state into that of any other, may be abbreviated by comparing the value of a dollar in one state with its value in any other: Thus, to reduce New-York money into Pennsylvania, subtract $\frac{1}{16}$ of the given sum from the same sum; since, 8s. New-York currency is equivalent to 7s. 6d. Pennsylvania, or 16 sixpences New-York money, is equivalent to 15 Pennsylvania; therefore, 16—$\frac{16}{16}$=16—1=15; that is, 1$\frac{1}{16}$*l*. New-York money is equivalent to 1*l*. Pennsylvania.

† The tables that precede most of the subdivisions of the remaining part of this article, must necessarily be employed in the work of almost every exercise. It may be a matter for the consideration of the teacher, whether they should be committed to memory by the pupil, or only referred to in the book when necessary.

Great Britain and Ireland.

194. The money of account is pounds, shillings, pence, and farthings; and the current coins are, by a late law passed in England, of the same value in both countries.

Table showing the par of Exchange with Great Britain.

$1·00	=4s. 6d. sterling.	£1 sterling	=$4·44$\frac{4}{9}$.
·22$\frac{2}{9}$	=1 shilling stg.	1 sovereign	=$4·44$\frac{4}{9}$.
·01$\frac{85}{100}$	=1d. sterling.	1 guinea or 21s.	=$4·66$\frac{2}{3}$.

Example 1. In £750 12s. British money, how many dollars?

s.	d.		£	s.		$		$
As 4	6	:	750	12	: :	1	:	3336.

Here, the terms being arranged according to the preceding rule, and then reducing the first and second terms to the same denomination, we shall have 54d. for the first term, and 180144 for the second; hence, 180144÷54=$3336, the answer.

In this example, the money of Great Britain is reduced to that of the United States at par; that is, at the rate of 4s. 6d. per dollar. It may be observed, however, that in procuring bills of exchange on Great Britain, a premium, which varies from 4 to 16 per cent., according to the state of trade, must be paid: for instance, if I want to procure a bill on London for £100 sterling, whose equivalent value in the United States, at par, is $444·44$\frac{4}{9}$, admitting the premium to be 10 per cent., I must pay $488·88$\frac{8}{9}$.*

* This premium, usually called the exchange, on British bills, depends principally on the importation of goods into the United States from England, and the exportation of the produce, &c. of the United States to that country. Whenever the imports from a country is greater than the exports to it, the bills of the foreign state must increase in value, as none of them will be required to be sent abroad to pay the balance; and thus the money in that country will increase in value, and will continue above par, till coin or bullion is remitted to make up the deficiency; or till, by a greater exportation or a smaller importation, or both, on the part of the debtor country, the equilibrium of trade is restored. It is of consequence to observe, that the course of exchange can never differ *very much* from the par, as coin or bullion will be remitted instead of bills, whenever the course of exchange is such, that the value of the coin or bullion, and the expense of remitting and insuring it would be less than the cost of bills. Besides, when the course of exchange is against a country, it affords an inducement to merchants

Exam. 2. A merchant in New-York owes a merchant in Liverpool £375 10*s.* 6*d.* sterling, how many dollars must the merchant in New-York pay for a bill to that amount, the premium being $12\frac{1}{2}$ per cent.?

s. d.	£ *s. d.*	\$ \$
As 4 6 :	375 10 6 : :	1 : 1669.

Here, the equivalent value of the bill at par, is \$1669; then, as \$100 : \$$112\frac{1}{2}$: : \$1669 : \$$1877{\cdot}62\frac{1}{2}$, the sum that the New-York merchant has to pay for a bill on Liverpool for £375 10*s.* 6*d.*

Exercises.—1. In \$7560·50, how many pounds sterling, the exchange being at par? Ans. £1701 2*s.* 3*d.*

2. How many pounds sterling are equivalent to one million of dollars, the exchange being at par?
Ans. £225000.

3. How many dollars are equivalent to one million pounds sterling, the exchange being at par? Ans. \$$4444444.44\frac{4}{9}$.

4. I owe a merchant in Dublin £250 4*s.* sterling, how much must I pay for a bill to that amount, the premium being $12\frac{1}{2}$ per cent.? Ans. \$1251.

5. How many pounds sterling must I remit to my correspondent in London, the sum I owe him being \$3000, and the premium is 14 per cent.? Ans. £769 10*s.*

6. How many dollars must a merchant in New-York receive for a bill on Liverpool, for £1000 10*s.* 6*d.* sterling, the premium being $13\frac{1}{2}$ per cent.? Ans. \$5047.09.2+.

Exchange with France.

195. Accounts are kept in France in *francs* and *centimes.* They were formerly kept in *livres*, *sous*, and *deniers.**

to export to the other country, as the bills they will get in return will be more valuable, in consequence of the money of the foreign country being above par; and thus they can procure a better and a surer market for their commodities, as they will be enabled to sell them at a lower price.

* The *livre*, *sous*, and *deniers*, were formerly used exclusively in accounts in France, but now the francs and centimes are employed, except in small transactions. The livre, also, had formerly the name of *franc;* but in a coinage during the time of the Republic, by some mistake in the mint, the five livre pieces were made too heavy, each being worth $101\frac{1}{4}$ sous, instead of 100, and the error made in the one coin was extended to the rest. The coin corresponding to the former livre

Table I.—10 centimes=1 decime; 100 centimes, or 10 decimes =1 franc.

Table II.—12 deniers=1 sous; 20 sous=1 livre; 3 livres=1 eur, or crown tornois; 81 livres=80 francs.

Gold coins of France, according to the laws of the United States, are rated at \$1 for $27\frac{1}{2}$ grains; and the par of exchange is usually reckoned at $18\frac{1}{2}$ cents; the relative value, however, of the money of France, as employed by the custom-house in determining the prices of goods, is $18\frac{1}{3}$ cents per franc, and at the rate of \$0·18173125 per livre tornois.

Ex. 3. Change \$1750.50 into French money, exchange at the rate of $18\frac{1}{3}$ cents per franc.

cts. \$ *cts.* *fr.* *fr. centimes.*

As $18\frac{1}{3}$: 1750 50 :: 1 : 9548 $18\frac{2}{11}$

Here, the first and third terms being reduced to the same denomination, (thirds of a cent,) we have 55 and 525150, the latter of which divided by the former gives 9548 francs, $18\frac{2}{11}$ centimes, the answer.

7. In 7560 francs, 90 centimes, how many dollars, the exchange being at 19 cents for 1 franc? Ans. \$1436.57.1.

8. Reduce \$2000 into French money at par.

Ans. 10810 francs, and $81\frac{3}{37}$ centimes.

Exchange with Spain.

196. In Spain there are two kinds of money called *plate* and *vellon;* and accounts are kept in both in *piastres*, *reals*, and *marvadies*. The *piastre* is also called the *pezza*, the *dollar of exchange*, and the *piece of eight*, or of $\frac{8}{8}$.

Plate money is more valuable than vellon, in the ratio of 32 to 17. Thus, 17 reals plate are equivalent to 32 reals vellon. Plate only is used in exchanges with England.

Hence, as 17 is to 32, so is any sum plate to the sum vellon equal to it; and as 32 is to 17, so is any sum vellon to the equivalent sum plate.

Table.—34 marvadies =1 real; 8 reals =1 piastre. Also 4 piastres =1 pistole of exchange; 375 marvadies =1

was called the *franc*, and the name *livre* appropriated exclusively to the old coin; and to facilitate calculations, the old division was laid aside, and the franc was divided decimally, as appears from the first of the above Tables.

ducat. The par of exchange with Spain is, 10 cents =1 real, plate; 5 cents =1 real, vellon: and the relative value of the gold coins of Spain are, in the United States, at the rate of $1 for 28½ grains.

Ex. 4. In 3000 piastres, 6 reals, plate, how many dollars?

	reals.		piastres	reals.		$		$	cts.
As	10	:	3000	6	::	1	:	2400	60

9. In $2000.50, how many piastres, &c. plate? Ans. 2500 piastres, 6¼ reals.

10. Reduce 1000 piastres, 7 reals, 30 marvadies, vellon, to dollars and cents. Ans. $800.78.8 $\frac{4}{17}$.

11. Reduce 1560 piastres, 6 reals, 20 marvadies, plate, to American currency. Ans. $1248.65.8 $\frac{14}{17}$.

Exchange with Amsterdam.

197. In Amsterdam, accounts are kept in florins, stivers, and pennings; and also in pounds, skillings, and grotes, or pence, Flemish.

Table —8 pennings =1 grote or penny, Flemish; 2 grotes or pence, or 16 pennings =1 stivre; 20 stivers, or 40 grotes =1 florin, or guilder; 6 florins =1 pound. Also, 12 grotes or pence =1 skilling; 20 skillings =1 pound. Also, 2½ guilders, or 50 stivres =1 rix dollar.

The par of exchange with Amsterdam is, 40 cents =1 florin, or guilder.

There are two kinds of money in Amsterdam, called *banco*, or bank money; and *currency*, or current money. In the former of these, all bills of exchange are valued and paid. It is of purer metal than the currency; and hence it bears a premium of 3 or 4, and sometimes 5 per cent.: that is, £100 of bank money is valued at £103, £104, &c. currency. This premium is called the *Agio*.

Ex. 5. In 250 florins, 10 stivers, and 12 pennings, how many dollars and cents?

	fl.		fl.	st.	p.		cts.		$	cts.
As	1	:	250	10	12	::	40	:	100	24

Here, the first and second terms must be reduced to the same denomination; that is, the florins ×20= stivers, stivers ×16= pennings; and then proceed as in simple proportion,

12. Reduce 2000 florins, 17½ stivres, to dollars and cents.
Ans. $800.35.

13. In $4000 how many florins, &c. of Amsterdam?
Ans. 10000 florins.

Exchange with Hamburgh.

198. Accounts are kept in Hamburgh in marks, schillings, and pfennings; and also in pounds, skillings, and pence, Flemish.

Table.—6 pfennings =1 grote, or penny, Flemish; 12 grotes or pence =1 skilling; 20 skillings =1 pound. Also, 12 pfennings, or 2 grotes or pence Flemish =1 schilling, Hamburgh money; 16 schillings =1 mark; 2 marks =1 dollar of exchange; and 3 marks =1 rix dollar. Hence, 1 skilling Flemish =6 schilling Hamburgh money, and the schilling is equal to the stiver.

In Hamburgh, as in Amsterdam, there are two kinds of money, *banco*, or bank money, in which exchanges are reckoned, and *current*. The agio on the former is high, varying from 18 to 25 per cent.

Par of exchange with Hamburgh, 33⅓ cents =1 mark banco.

14. Reduce 5127 marks, 5 schillings, Hambro' money, to dollars and cents. Ans. $1709.10.4⅙.

15. In $756.50, how many marks, &c. Hambro' money?
Ans. 2269½ marks.

Exchange with Portugal.

199. In Portugal, accounts are kept in *milrees* and *rees*.

Table.—1000 reas =1 milree. Also, 400 rees =1 crusado, and 4800 rees =1 moidore: consequently, 2½ crusadoes =1 milree.

Par of exchange with Portugal—$1.24=1 milree.

16. In $2000, how many milrees and rees?
Ans. 1612 milrees, 903$\frac{37}{124}$ rees.

17. In 2000 milrees, 500 rees, how many dollars and cents? Ans. $2480.62.*

* If the pupil be made fully acquainted with what precedes, respecting exchanges, he will find little difficulty, with the assistance of tables,

18. Reduce 2467 pezzi, 12 soldi, 6 denari, of Leghorn, to American money, exchange at $87\frac{1}{2}$ cents per pezza.
Ans. $2159.17.$1\frac{7}{8}$.

19. Reduce $300 to pezzi, &c. of Leghorn, exchange at 85 cents per pezza. Ans. 352 pezzi, 18 soldi, $9\frac{14}{17}$ denari.

20. Reduce 1200 ducats regno, 8 carlins, 9 grains, Naples currency, to dollars and cents, exchange at 75 cents per ducat regno. Ans. $900.66.$7\frac{1}{2}$.

21. Reduce $1250.75 to ducats regno, &c. Naples currency; exchange at 74 cents per ducat regno.
Ans. 1690 ducats regno, $2\frac{2}{37}$ carlins.

in applying the same principles to similar cases, which it would exceed the limits of the present publication to illustrate individually. To facilitate this the following tables are annexed, which will be found to contain what is most useful and necessary on the subject.

LEGHORN.—12 denari di pezza =1 soldo di pezza; 20 soldi di pezza =1 pezza of 8 reals. Also, 12 denari di lira =1 soldo di lira; 20 soldo di lira =1 lira; $5\frac{3}{4}$ lira, moneta-buona, =1 pezza of 8 reals.

Par of exchange with Leghorn, 1 pezza of 8 reals =$86\frac{5}{8}$ cents.

GENOA.—The same table as for Leghorn serves for Genoa; besides, 4 lire and 12 soldi =1 scudio di cambio, or crown of exchange; 10 lire and 14 soldi =1 scudio d'oro marche, or gold crown. Pezza, or dollar of exchange, =85 cents, nearly.

NAPLES.—10 grains =1 carlin; 10 carlins, or 100 grains =1 ducat regno; 1 ducat regno =$75\frac{1}{2}$ cents, nearly.

VENICE.—12 denari =1 soldo; 20 soldi =1 lira; 6 lire and 4 soldi =1 ducat current, or of account; 8 lire =1 ducat effective; 1 lira peccola, new coin, =$7\frac{9}{16}$ cents, nearly.

PETERSBURG.—100 copecs =1 ruble; 1 ruble =72 cents.

VIENNA.—4 pfennings =1 creutzer; 60 creutzers =1 florin; 90 creutzers, or $1\frac{1}{2}$ florins =1 rix dollar of account; 1 florin =$66\frac{1}{2}$ cents, nearly.

STOCKHOLM.—12 fenings, or oers, =1 skilling; 48 skillings =1 rix dollar =1 dollar United States' money.

COPENHAGEN.—12 pfenings =1 skilling; 16 skillings =1 mark; 6 marks Danish, (or 3 marks Hambro') =1 rix dollar =1 American dollar.

The East India coin which is most frequently mentioned is the *rupee*. Of this there are two kinds, the *current rupee* and the *sicca rupee*. The value of the former is about 39 cents, or 21*d*. sterling; and that of the latter 45 cents, or $24\frac{1}{2}$*d*. sterling, or more accurately, $45\frac{1}{2}$ cents, and 24.566*d*. sterling. The current is to the sicca rupee as 100 is to 116. In India, the market price of rupees is generally much higher than their intrinsic value, the current rupee being worth about 2*s*. and the sicca rupee 2*s*. 6*d*. sterling.

The following exercises, which the pupil will find to be easily resolved, will serve to illustrate these tables.

Arbitration of Exchanges.

200. When the courses of exchange between the first and the second, the second and the third, the third and the fourth, &c. of any number of places, are given, the method of finding the course of exchange between the first place and the last, corresponding to these courses, or of the valuing of any sum of the money of the first place in that of the last through the medium of the others, is called *Arbitration of Exchanges.**

201. All the operations may be performed by one or more analogies in the rule of proportion.

The method by the rule of proportion is easy and intelligible, when there are only three places concerned, or in what has been termed *simple arbitration.* But when more than three places are concerned, or what has been called *compound arbitration*, the following rule, usually called the *chain rule*, is generally preferable.

202. Rule.—Let all the quantities of the same kind be reduced to the same denomination, if they be not so already. Let a blank† be left for the required quantity, and to the right of it place as *consequent* the term to which it is to be equivalent: then, below the blank, place as *antecedent* the other given term which is of the same kind as the last consequent, and to the right of it place as consequent the term which is equivalent to it. Proceed thus, till the terms are arranged in two vertical columns: then, divide the continual product of the

* As the actual course of exchange between the first place and the last is almost always, from various circumstances, different from the arbitrated course, this method is of use in enabling a merchant in one place to discover whether he should draw and remit directly between his own place and another, or circuitously through other places.

† The interrogation mark (?) may properly be placed in the blank.

This rule will be found considerably more easy in its application than those usually given for the same purpose in books on arithmetic. It is also expressed in general terms, so as to serve not only for the purpose of exchange, but for the comparison of weights and measures, and for any other uses to which it can be applied.

consequents by the continual product of the antecedents, and the quotient will be the result required.

The operation may often be much abridged by striking out any antecedent and consequent that are equal, and by reducing to their lowest terms such as admit a common divisor.

Exam. 1. When the exchange between England and America is at par, and between England and Amsterdam at 36*s*. 4*d*. Flemish per pound sterling, what is the arbitrated rate of exchange between America and Amsterdam ?

Flem. ? = 1 dollar
$1 = 54*d* sterling
Eng. 240*d*. = 436*d*. Flemish.

$$\frac{54\times436}{240}=\frac{9\times109}{10}=98\tfrac{1}{10}d.=8s.\ 2\tfrac{1}{10}d.$$

Here, the American money is in dollars, and the value of a dollar in English money being 4*s*. 6*d*., this being reduced to pence, the pound must also be reduced to pence : then the Flemish being reduced to pence, the answer is found in pence, Flemish, and becomes, by reduction, 8*s*. $2\frac{1}{10}d$. Flemish, per dollar.*

21. What is the value of a franc in American money, exchange between England and America at par, and between England and France at 24 francs, 87 centimes per pound sterling? Ans. $\$0{\cdot}17{\cdot}8\frac{15826}{22383}$.

22. Suppose a merchant in England owes a merchant in Portugal £572 : whether is it better for the Portuguese merchant to have a direct remittance from London to Lisbon at 68*d*. per milree, or a circular remittance through Amsterdam and Paris, exchange between London and Amsterdam being 37*s*. 3*d*. Flemish per pound sterling ; between Amsterdam and Paris at 56*d*. Flemish for 3 francs ; and be-

* This question might have been solved by a single analogy. Thus, as 240*d*. : 54*d*. : : 436*d*. : $98\frac{1}{10}d$. In this mode, we multiply and divide by the same quantities as in working by the chain rule ; and the same is the case in all applications of this rule. This not only shows the correctness of the rule, but also that it is nothing else than simple or compound proportion exhibited in a different, and, in many cases, in a more convenient form. The learner will perceive, that in the arrangement of the terms by the chain rule, the first antecedent and the last consequent are always of the same kind.

tween Paris and Lisbon at 460
of 1½ per cent. being incurred
Ans. The circular exchange
to the Portuguese merchant, as
rees, 933 rees more than by th
23. Suppose that a merchan
in London 12000 rubles, and th
tween London and Petersburg
required to find how much mo
the London merchant to draw d
draw through Paris, Amsterda
the course of exchange betwe
24 francs, 55 centimes per po
and Amsterdam at 55 grotes Fl
Amsterdam and Hamburgh at 3
bro'; between Hamburgh and
Hambro' for 299 rix dollars c
enna and Petersburgh at 185 c
Ans. The direct remittance

CHAPTE

SERIE

203. A *series* is a successi
that depend on one another.
In every series the first an
extremes, and the rest the *me*

Writers on arithmetic usuall
series, *equi-different series* a
These are of more frequent
kinds of series, and on this ac
attention. Quantities in equi-
in *arithmetical progression;* an
said to be in *geometrical progre*

* These names for equi-different
tionals are highly improper. Series
arithmetic and geometry. The appe
geometrical progression, should, theref
to impress false ideas on the mind, r

Equi-different Series.

204. An equi-different series is that in which the terms all increase, or all decrease, by the same quantity: this quantity is called the common difference.

Thus, 5, 7, 9, 11, 13, is an equi-different series, in which the successive terms increase by the addition of the common difference 2: and 20, 17, 14, 11, 8, is another, in which the successive terms decrease by the common difference 3.

The following are the most useful rules for the management of quantities of this kind.

PROBLEM I.—*The first term and the common difference being given, to find any other assigned term.*

205. RULE.—Multiply the common difference by the number which is equal to the number of the terms preceding the required term: then if it be an increasing series, add the product to the first term; otherwise, subtract it.

Example 1. Required the thirty-fifth term of the increasing equi-different series, whose first term is 7, and common difference 3.

Here, 34 precede the required term; wherefore, $34\times3+7=109$, is the term required.*

Exercises.—1. Required the fifty-fourth term of the decreasing equi-different series, whose first term is 100, and common difference $1\frac{1}{4}$. Ans. $33\frac{3}{4}$.

2. First term of an increasing series 36, common difference $3\frac{3}{5}$; required the hundredth term. Ans. $392\frac{2}{5}$.

3. First term of a decreasing series 329, common difference $\frac{7}{8}$; required the ninety-ninth term. Ans. $243\frac{1}{4}$.

PROBLEM II.—*The extremes and the number of terms being given, to find the sum of the series.*

206. RULE.—Multiply the sum of the extremes by the number of terms, and take half the product.

* *The reason of this operation* will be manifest from the consideration, that were the series to be continued to the thirty-fifth term, the first term must be increased by 34 additions of the common difference.

Exam. 2. The first term of an equi-different series is 1, its last term 312, and the number of terms 193. What is its sum?

Here, $1+312=313$, and $313\times193=60409$, the half of which is $30204\frac{1}{2}$, the sum of the series.*

Ex. 4. Given the greater extreme $=1000$, the common difference $=2\frac{1}{6}$, and the number of terms 367; required the sum of the series. Ans. $221484\frac{1}{2}$.

5. Given the greater extreme $=1$, the common difference $=\frac{1}{100}$, and the number of terms $=51$; required the sum of the series. Ans. $38\frac{1}{4}$.

Problem III.—*The extremes and the common difference being given, to find the number of terms.*

207. Rule.—Divide the difference of the extremes by the common difference, and add a unit to the quotient.†

Ex. 6. Given the greater extreme $=500$, the less $=70$, and the common difference $=10$; required the number of terms. Ans. 44.

7. Given the less extreme $=3$, the greater $=579$, and the common difference $=9$; what is the sum of the series? Ans. 18915.

8. With the common difference 12, how many equi-different means can be inserted between the extremes 8 and 1700? Ans. 140.

* *The reason of this rule* will be understood from the following property of equi-different quantities:—In any equi-different series, 5, 8, 11, 14, 17, 20, 23, the sum of 5 and 23 is equal to the sum of 8 and 20, and of 11 and 17, and is double of the middle term 14. The reason of this will appear manifest, if it be considered that 8 and 11 respectively exceed the less extreme by the same quantities by which 20 and 17 are respectively less than the other extreme. Hence, with respect to the sum of this latter series, it is evident that though each term were made 14, half the sum of the extremes, still the *sum* of the whole would be the same; and, consequently, the sum of the series will be obtained by multiplying half the sum of the extremes by the number of terms, or, which is the same, by multiplying the sum of the extremes by the number of terms, and taking half the product.

† The reasons of this rule and of the next will be obvious from comparing them with Rule, Prob. I.

PROBLEM IV.—*The extremes and the number of terms being given, to find the common difference.*

208. RULE.—Take a unit from the number of terms, and by the remainder divide the difference of the extremes.

Ex. 9 Given the extremes =3 and 300, respectively, and the number of terms =10 ; required the common difference. Ans. 33.

10. What is the common difference of a series, consisting of 1001 terms, the extremes being 1 and 100001 ? Ans. 100.

PROBLEM V.—*The extremes being given, to find any assigned number of equi-different means.*

209. RULE.—Find the common difference by Rule IV. and add it continually to the less extreme, or subtract it from the greater; the several results will be the required terms. One mean may be found by taking half the sum of the means.

Exam. 3. Given the first term =1, the last =99, and the number of terms =8 ; required the complete series.

By Rule IV. the common difference is found to be 14, by the continual addition of which to 1, the entire series is found to be 1, 15, 29, 43, 57, 71, 85, 99.

Ex. 11. Insert 5 equi-different means between 20 and 30. Ans. $21\frac{2}{3}$, $23\frac{1}{3}$, 25, $26\frac{2}{3}$, $28\frac{1}{3}$.

12. Required the several terms of a series, the extremes of which are 4 and 49, and the number of terms 6. Ans. 4, 13, 22, 31, 40, 49.

PROBLEM VI.—*The sum of the series, one extreme, and the number of terms being given, to find the other extreme.*

210. RULE.—Divide twice the sum of the series by the number of terms, and from the quotient take the given extreme.

The reason of this rule is evident from Rule II.

Ex. 13. Given the first term of an equi-different series, consisting of 24 terms =1 ; required the last term, the sum of the series being =576. Ans. 47.

14. Given the number of terms =50, and the sum of the series =1275; required the greater extreme, the less being $3\frac{3}{4}$. Ans. $47\frac{1}{4}$.

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15. Required the sum of the first ten thousand numbers, in the natural series 1, 2, 3, 4, 5, &c. Ans. 50005000.

16. Required the sum of the first ten thousand odd numbers, 1, 3, 5, 7, &c. Ans. 100000000.

17. Required the sum of the first ten thousand even numbers, 2, 4, 6, 8, &c. Ans. 100010000.

18. If a person on a journey travel the first day 30 miles, and each succeeding day a quarter of a mile less than he did the day before, how far will he travel in 30 days?

Ans. $791\frac{1}{4}$ miles.

19. How many strokes does a common clock strike in the year? Ans. 56940.

20. A body falling by its own weight, if it were not resisted by the air, would descend in the first second of time through a space of 16 feet and 1 inch; in the next second, through 3 times that space; in the third, through 5 times that space; in the fourth, through 7 times, &c. Through what space would it fall at the same rate of increase in a minute? Ans. 57900 feet.

Continual Proportionals, or Geometrical Progressions.

211. *A series of continual proportionals* is that in which the successive terms all increase by a common multiplier, or all decrease by a common divisor. The common multiplier, or common divisor, is called *the ratio of the series*, or *the common ratio.*

Thus, 3, 6, 12, 24, 48, are continual proportionals, in which the successive terms increase by the ratio 2; and 192, 48, 12, 3, $\frac{3}{4}$, &c. are continual proportionals, decreasing by the ratio 4.*

The following are the most usual rules for managing quantities of this kind.

* It might be observed, that we might regard every series of this kind, whether increasing or decreasing, as being produced by multiplication, the ratio in a decreasing series being a proper fraction. Thus, in the series last given, the ratio, or common multiplier, might be considered to be $\frac{1}{4}$. In what follows, however, the ratio will be taken always greater than a unit, according to the definition already given.

Problem I.—*The first term and the ratio being given to find any other proposed term.*

212. **Rule.**—Raise the ratio to a power whose index is equal to the number of terms which precede the required term: then, if it be an increasing series, multiply the first term by the result before found; otherwise divide it by that result.

Example 1. Required the 8th term of the series of continual proportionals, whose first is 6, and ratio 2.

Here, the 7th power of 2 is found to be 128; which being multiplied by the first term 6, the product is 768, the 8th term.*

Ex. 2. Required the 20th term of the series, whose first term and ratio are each 1.06.

Here, we are to multiply the 19th power of 1.06 by 1.06, or, which is the same, we are to involve 1.06 to the 20th power. This is found by involution to be 3.207135.

Ex. 3. Required the sixth term of the decreasing series, whose first term is 100, and ratio $1\frac{1}{2}$.

The 5th power of $1\frac{1}{2}$, or 1.5, is 7.59375, and 100 being divided by this, the quotient is 13.16872428, the term required.

Exercises.—1. Given the first term of an increasing series 12, and its ratio 3, to find the 18th term. Ans. 1549681956.

2. Given the first term of a decreasing series =500, and the ratio =1.04; to find the 14th term. Ans. 300.287.

3. The first term of a decreasing series is 1, and the ratio 1.07; required the 14th term. Ans. .4149644.

4. What did the last of 12 oxen cost, the first of which was sold for $3, the second for $9, and so on?

Ans. $531441.

* *The reason of this operation* will be manifest if it be considered, that in finding the successive terms up to the 8th, the first term must be multiplied by 2, the product by 2, that product by 2, and so on, till the 8th term would be found after 7 such multiplications; and it is evident, that the same result will be found by a single multiplication by the 7th power of 2. A similar illustration serves in case of a decreasing series.

PROBLEM II.—*To find the sum of a series of continual proportionals.*

213. RULE.—Multiply the greater extreme by the ratio, and divide the difference between the product and the less extreme, by the difference between the ratio and a unit.

When the series is a decreasing one, and the number of terms infinite: divide the product of the ratio and the greatest term by the difference between the ratio and a unit; or, divide the ratio by the difference between it and a unit, and multiply the quotient by the first term.

Ex. 4. Given the first term of an increasing series $=4$, the ratio $=3$, and the number of terms $=6$; to find the sum of the series.

Here, by Rule I. we find the last term to be 972. Multiplying this by the ratio, we obtain 2916: and dividing 2912, the difference between this and the first term, by 2, the difference between the ratio and a unit, we obtain 1456, the required sum.*

Ex. 5. Required the value of the interminate decimal .1′8′.

This is the same as $\frac{18}{100}+\frac{18}{10000}+\frac{18}{1000000}+$, &c. continued without limit, where the ratio is evidently 100. Multiply the first term, therefore, by 100, and dividing the result by 100—1, we obtain for the sum of the series, or the value of the decimal, $\frac{18}{99}$, or in its lowest terms $\frac{2}{11}$.

* *The reason of the operation* is best shown by algebra; it may be illustrated, however, in the following manner: let the terms of the series be placed as in the margin; then, let each term be multiplied by the ratio, and the products be removed each one place to the right hand.

$$\begin{array}{l} 4+12+36+108+324+972= \text{sum}, \\ \quad\;\; 12+36+108+324+972+2916= \text{sum} \times 3. \end{array}$$

If the upper line be then subtracted from the lower, there will remain $2912=$ sum $\times 2$; and, consequently, the sum is equal to $2912\div2=1456$. Now, 2916 is evidently the product of the ratio and the greater extreme, and 2912 is the difference between this and the less extreme; also, the divisor 2 is the difference between the ratio and a unit; and a similar illustration may be given in any other case. In a decreasing infinite series, the last term is to be regarded as nothing; and hence the reason for its summation is manifest. For a full illustration of these rules, the reader is referred to my *Treatise on Algebra*, Art. 487.

5. Given the first term of an increasing series $=6$, the ratio $=4$, and the number of terms $=8$; to find the sum of the series. Ans. 131070.

6. Find the sum of the infinite series whose greatest term is 100, and ratio 1.04. Ans. 2600.

7. Find the sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c.
Ans. 1.

8. Find the sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c.
Ans. $\frac{1}{2}$.

9. Find the value of $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$, &c. *ad infinitum.*
Ans. 2.

10. Find the value of $1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+$, &c. *ad infinitum.*
Ans. 4.

11. A gentleman, who had a daughter married on New-Year's day, gave the husband towards her portion \$4. promising to triple that sum the first day of every month, for nine months after the marriage: the sum paid on the first day of the ninth month was \$26244. What was the lady's fortune? Ans. \$39364.

CHAPTER XV.

Equation of Payments, Compound Interest, and Annuities.

Equation of Payments.

214. When one person owes another several debts, payable at different times, the rule which determines the just time for a single payment of the whole, is called *equation of payments.*

Rule I.—Multiply each debt by the time that must elapse before it will become due; then divide the sum of the products thus obtained by the sum of the debts, and the quotient will be the time required.*

* The rule above given is that which is generally preferred in working questions in equation of payments. No rule in arithmetic, however, has given origin to so warm disputes; some writers arguing strongly in support of its accuracy, the others entirely condemning the principles

When the products are taken, all the times, and likewise all the debts must evidently be in the same denomination.

Example 1. If one person owe to another \$300 payable in 4 months, \$500 payable in 6 months, and \$400 payable in $10\frac{1}{2}$ months; in what time might the whole debt be paid without loss to either party?

Here, $300\times4+500\times6+400\times10\frac{1}{2}=8400$, and $300+500+400=1200$; then, $8400\div1200=7$. Hence, the time required, according to the preceding rule, is 7 months.*

Ex. 2. If a person owe \$100 payable at present, and \$600 payable in 7 months, at what time may both be justly paid at a single payment?

Here, $100\times0+600\times7=4200$, and $100+600=700$; then, $4200\div700=6$, the months required.

Exercises.—1. Required the equated time for the payment of two debts, one of \$350 due at the end of 8 months, and another of \$600 due at 13 months. Ans. $11\frac{3}{19}$ months.

2. If a debt of \$45 be payable at 6 months, another of \$70 at 11 months, and a third of \$75 at 13 months; what is the equated time for the payment of the whole?

Ans. $10\frac{23}{38}$ months.

3. If a debt of \$1200 be payable, one-half at 18 months, one-fourth at 15 months, one-sixth at 10 months, and the remainder at 3 months, what is the equated time for the payment of the whole? Ans. $14\frac{2}{3}$ months.

4. What is the equated time for the payment of four debts, the first for \$120 due at 1 month, the second for \$135

from which it is deduced. To renew the dispute on this unimportant subject, and to echo the arguments that have been advanced on both sides of the question, is inconsistent with the plan of this work. It may suffice to say, that the principle on which this rule depends is, that the interest of the money, the payment of which is delayed beyond the time at which it is due, is to be equal to the interest of that which is to be paid before it becomes due, and that this principle is liable to the same objections as the common rule for discount.

* The work may be proved according to the principle on which the preceding rule is founded, by finding the interest of the first debt at any assumed rate for 3 months, the interest of the second at the same rate for 1 month, and the interest of the third at the same rate still for $3\frac{1}{2}$ months: the last of these interests will be found to be exactly equal to the sum of the other two. The times used in the proof are the differences between the equated time and the times at which the several debts are due.

due at 7 months, the third for \$160 due at 10 months, and the fourth for \$100 due at 1 year? Ans. $7\frac{52}{103}$ months.

5. What is the just time for the payment of two debts, the one of \$2786 due at the end of 7 months, and the other of \$3412.50 due at the end of $2\frac{3}{4}$ years?

Ans. $21\frac{556}{1771}$ months.

6. If a debt be payable one-third at present, one-fourth at 6 months, one-fifth at 12 months, and the remainder at 18 months, what is the equated time for the payment of the whole? Ans. $7\frac{3}{5}$ months.

215. Rule II.—Find the present worth of the several debts at the given or common rate of interest: then find, by Rule V. page 187, in what time, at the same rate, the sum of the present worths thus found would amount to the sum of the debts. The result thus obtained will be the time required.*

The following are the answers of the exercises according to the second rule; the first at 5, and the rest at 6 per cent. per annum:—

1. 11 months, 4 days.	4. 7 months, 13 days.
2. 10 months, 17 days.	5. 20 months, $16\frac{1}{2}$ days.
3. 14 months, 17 days.	6. 7 months, 17 days.

Compound Interest.

216. Rule I.—Find the amount of the given principal for the time of the first payment by simple interest. Consider this amount as the principal of the second payment, the amount of which is calculated as before, and so on through all the payments to the last, still

* In this rule, the result depends on the *present* values of the debts; and it appears to be an obvious principle, that all such transactions and agreements should be regulated in conformity to the *present value* of the money, and the *improvement* of which it is susceptible. This latter principle, which is entirely neglected in the use of the first rule, is acted on in the second, in using the rate of interest. When the times are short, however, the difference of the results by the two rules is small, and the first being easier in its application, may be employed with as much propriety as the common rule for discount. For farther information upon this subject, the curious reader is referred to *Malcolm's Arithmetic*, page 621, where he will find another rule, the principles of which derived from the consideration of interest and discount.

reckoning the last amount as the principal for the next payment.*

Example 1. What is the amount of $4000 for 3 years, at 5 per cent. compound interest?

Here, the interest is one-twentieth of the principal; hence this method:

	$	
20)	4000	given principal,
	200	first year's interest,
20)	4200	second year's principal,
	210	second year's interest,
20)	4410	third year's principal,
	220.50	third year's interest,

Answer $4630.50, amount.

From the last amount subtract the given principal, and the remainder will be the compound interest. Thus, $4630.50—$4000=$630.50, compound interest.

Exercises.—1. What is the amount of $400 for 4 years, at 6 per cent. compound interest? Ans. $504.99.

2. What is the amount of $600 for 5 years, at $4\frac{1}{2}$ per cent. compound interest? Ans. $747.70.7.

217. RULE II.—*To find the amount of $1 for any number of years at compound interest.* Divide the amount of $100 for 1 year by 100, and the quotient will be the amount of $1 for 1 year: this amount involved to a power denoted by the number of years, will be the amount of $1 for that number of years.

Exam. 2. What is the amount of $1 for 2 years, payable half yearly, at 6 per cent. per annum?

Here, the payments being *half yearly*, the amount of $100 for half a year is $102·50; and consequently, that of $1 at the same time is $1·02·5; the square of this is $1·050625, the amount for two half years, or one year. Multiply this by 1·050625, we shall find the amount at the end of two years; that is, $1·1038129.

* When the time is short, this method may be practised without much trouble; but when the time is long, the labour would become very great. In this case, the methods pointed out in Rules II. and III. should be employed.

218. RULE III.—*To find the amount, or the interest, of any sum, at compound interest, for a given time, and at a given rate:* Find the amount of \$1 for the given time by Rule II. and multiply it by the given sum; the product will be the amount required.

If the principal be subtracted from the amount, the remainder will be the interest.

Exam. 3. Required the amount of \$1000 for 12 years, at 5 per cent. per annum, compound interest.

By Rule II. the amount of \$1 for the given time is found to be \$1.795856. This being multiplied by 1000, the result will be \$1795.85.6, the amount required.

3. What is the compound interest of \$2500 for 5 years, at 6 per cent. per annum? Ans. \$3345.56.3.

4. What is the amount of \$8750 for 4 years, at 5 per cent. compound interest? Ans. \$10635.67+.

219. RULE IV.—*To find the principal, which, at a given rate, and in a given time, will amount to a given sum:* Or, *to find the present worth of a sum at compound interest for a given time.* Divide the given sum by the amount of \$1, found by Rule II., and the quotient will be the principal or present worth required.

The present worth of \$1 may be found by dividing it by its amount for the given time.

Exam. 4. What sum must be lent at compound interest, at 5 per cent. per annum, at the birth of a child, so that the amount may be \$3000 at the end of 21 years?

Here, the amount of \$1 for 21 years, found by Rule II., being 2.785962, we have for answer \$3000÷2.785962= \$1076·90 nearly.*

* *The reason* of the second rule will appear from the following consideration. The amount of \$1 for a year will evidently be a hundredth part of the amount of \$100: and as \$1 is to its amount for a year, so is any other principal to its amount for the same time. Hence, to take a particular instance, the amount of \$1 for a year at 5 per cent., will be \$1·05; and by the nature of compound interest, this will be the principal for the second year. Then, as the principal \$1 : \$1·05, its amount : : the principal \$1·05 : (\$1·05)^2, which will be the amount at the end of the second year, and the principal for the third year. In this manner it will appear, that the amount of one dollar for any number of years will be equal to \$1·05 raised to the power denoted by the number of years. The amount of \$1 being thus determined, it is evident that the amount of any other principal will be had by multiplying the amount of \$1 by that principal, since the amount will evidently be proportional to the principal, which proves the rule. The fourth rule is evidently the converse of the third, and hence its correctness is evident.

Ex. 5. A brother is to pay his sister a portion of \$4500, at the end of 11 years: how much will discharge the debt at the end of 4 years, compound interest being allowed at 4 per cent. per annum on the sum he pays? Ans. \$3419·63.

6. With what capital must a merchant commence trade, to be worth \$15000 at the end of 12 years, if he may be expected to clear annually an eighth of his capital?

Ans. \$3649·73.

220. It may be proper to observe, that if interest be payable *yearly*, the amount of \$1 at the end of six months will be the square root of its amount for a year; its amount for 4 months, or one third of a year, the cube root of the same, for 3 months its fourth root, or the square root of the square root, &c.*

Ex. 7. If a boy 12 years old, has a legacy of \$1396$\frac{3}{4}$ left to him, how much will he have to receive at the age of 21, the legacy being improved at compound interest at 5 per cent. per annum? Ans. \$2166·68.8.

* Farther also, the amount of \$1 for 2 days will be the square, for 3 days the cube, &c. of its amount for one day; its amount for 8 months will be the square of its amount for 4 months; its amount for 9 months the cube of its amount for 3 months; and, finally, its amount for a year and a quarter will be the product of its amount for a year, and for three months; and its amount for 6 years and a half, will be its amount for 6 years, and for half a year; and so on in similar cases. All this will appear obvious from a due consideration of the nature of compound interest.

If the interest were payable half yearly, however, at a given rate per cent. per annum, as in the 4th example, the amount at the end of a year would be more than if it were payable yearly. Thus, at 4 per cent. per annum, payable half yearly, the amount at the end of half a year would be \$1·02; at the end of a year, or two half years, \$$1{\cdot}02^2$, or \$1·0404; at the end of a year and a half, or three half years, \$$1{\cdot}02^3$, &c. Had it been employed payable quarterly, the amounts at the end of 1, 2, 3, &c. quarters would have been \$1·01, \$$1{\cdot}01^2$, &c. In such cases, *to find the amount of* 1 *dollar at the end of the proposed time, raise its amount at the end of the first payment, to the power denoted by the number of payments.*

As calculations in Compound Interest are much facilitated by the use of interest tables, a table constructed by Rule I. is given at the end of the book, showing the amount of one dollar at compound interest for any number of years not exceeding 50, at the most useful rates. The pupil after having wrought the preceding exercises, by the rules already given, should be taught, instead of finding the amount of \$1 by Rule I. to take it where he can from the table. This table will also be useful in finding, by inspection, in many instances, the time, from the principal, rate, and amount; and the rate from the principal, time, and amount.

8. Required the amount of $5000, from the 12th till the 18th of a young lady's life, at 6 per cent. per annum, compound interest. Ans. $7092·59½.

9. Required the amount of $500, from the 6th till the 21st of a boy's life, at 5 per cent. per annum, compound interest. Ans. $1039·46·4.

10. Whether is it better to sell a farm for $1000 payable at present, $1000 payable at the end of 5 years, and $1000 payable at the end of 10 years; or to sell it for $3000 payable at the end of 5 years, compound interest being allowed at 4 per cent. per annum?

Ans. Better at three payments, by $31·71½.

ANNUITIES.

221. *An Annuity* is a fixed sum of money payable at the end of equal periods of time; such as years, half years, and quarters. Annuities are of two kinds, *certain* and *contingent*.

222. *Annuities certain* are those which commence at a fixed time, and continue for a determinate number of years.

223. *Annuities contingent* are those whose commencement, or continuance, or both, depend on some contingent event, usually the life or death of one or more individuals.

224. The *present* value* of an annuity at compound interest, is such a sum as would, if lent at compound interest for the given time, amount to the same sum to which the annuity itself would amount, if forborn during the same time.

When an annuity does not come into possession till a given time has elapsed, or some particular event has taken place, it is said to be *an Annuity in Reversion*.

* An annuity is commonly said to be worth as many *years' purchase* as there are dollars in the present value of an annuity of $1. Thus, in the case of an annuity for 20 years, at 5 per cent. per annum, because the present value of an annuity of $1 is $12·462, &c. is said to be worth about twelve and a half year's purchase.

Annuities Certain.

PROBLEM I.—*To find the amount of an annuity, payable yearly, the payments of which are forborn for a given time, compound interest being charged on them as they become due.*

225. RULE.—Subtract a unit from the amount of $1 for a year, and from its amount for the given time at compound interest: divide the latter remainder by the former, and the quotient will be the amount of an annuity of $1 forborn for the proposed time: multiply this amount by the given annuity, and the product will be the amount required.

When the payments are not yearly, instead of the amount of $1 for a year, use its amount for the interval between the payments; and instead of the number of years, use the number of payments that would have been made during the time they were remitted, and then proceed as before.

Example 1. If a person save $120 per annum, and improve it at 5 per cent. per annum, compound interest, how much will he be worth at the end of 20 years?

The amounts of $1 for 1 year and for 20 years, at 5 per cent. per annum, are (as found by Rule I. Compound Interest) 1·05 and 2·6532977; from each of which if a unit be subtracted, there remains ·05, and 1·6532977. Let the latter of these be divided by the former, and the quotient, 33·065954, is the amount of an annuity of $1 for 20 years; then let this be multiplied by 120, and the product, $3967·91448, or $3967·91½ nearly, is the amount required. In this case, the gain by interest is $1567·91½, since the person's savings without interest, would have been $120 × 20, or $2400.*

* The theory of this rule is much more easily and satisfactorily explained by Algebra investigation, than it can be by common Arithmetic. For the use of those, however, who are unacquainted with Algebra, the following illustration of a particular case is annexed. Let it be required to find the amount of an annuity of $1 for 8 years, at 5 per cent. per annum, compound interest. At the end of the time the eighth payment would be simply $1; the value of the seventh would be $1·05, as it would remain at interest 1 year: that of the sixth $$1·05^2$, as it would remain at interest 2 years: that of the fifth $$1·05^3$; of the fourth $$1·05^4$; of the third $$1·05^5$; of the second $$1·05^6$; of the first $$1·05^7$. Hence, the entire amount to be received at the end of the time would be the sum of the series 1, 1·05, $1·05^2$, $1·05^3$, $1·05^4$, $1·05^5$, $1·05^6$, $1·05^7$

Exam. 2. Let every thing be as in the last example, except that the annuity is payable *half yearly*, instead of *yearly*.

Here, since the payments are half yearly, there would have been 40 payments; and the amount of \$1 at the end of half a year in these circumstances, is \$1·025, the 40th power of which is 2·6850723, the amount of \$1 at compound interest at the end of 20 years. Then 1·6850723÷·025=67·402892, is the amount, at the end of 20 years, of an annuity of \$1 payable at the end of each period of 6 months. Multiply this by 60, the sum payable each half year, and the product, \$4044·17352, or \$4044·17, nearly, is the amount required, which is \$76·25½ more than the answer of the last question. It is evident, that the more frequent the payments are, the greater is the amount: for the several gains by interest are thus put sooner to gain more interest.*

Exercises.—1. If a person rent a farm at \$120 per year, payable yearly, and forbear paying rent for 16 years; how much will he owe to the proprietor at the end of that time, allowing him compound interest at 5 per cent. per annum?
Ans. \$2838·89·9.

2. Suppose a person who has a salary of \$750 a year, payable yearly, to allow it to remain unpaid 17 years: how much will he be entitled to receive at the end of that time, compound interest being allowed at 6 per cent. per annum? Ans. \$21159·66.

But by Rule II. page 249, the sum of this series is $(1·05^8-1)÷·05$, which agrees with the rule here given for finding the amount of an annuity of one dollar. The rest is obvious.

It may serve to illustrate the nature of annuities, to show another method of resolving the above example, which method might also be employed in solving all questions of a similar nature. Thus, as \$5 : \$100 : : \$120 : \$2400, the principal which would gain \$120 per annum. Then at compound interest, the amount of \$2400 for 20 years is \$6367·91½, from which \$2400 being subtracted, we have remaining \$3967·91½ for interest, or improvement of this imaginary principal, which is also the amount of the annuity.

* When the pupil shall have learned to perform the exercises on this rule and the next, he may be taught to use Tables II. and III. at the end of the book as often as they are applicable. By this means the labour will often be greatly abridged, in the same manner as operations in Compound Interest are often much shortened by the use of Table I.

PROBLEM II.—*To find the present value of an annuity at compound interest.*

226. RULE.—Find by the last rule the amount of an annuity of $1 for the given time, and at the given rate: divide this by the amount of $1, at compound interest for the given time, and the quotient will be the present value of an annuity of $1 for the given time: multiply this by the annuity to find the present value required.

In case of an annuity to continue for ever, or, as it is called, *a perpetuity*, subtract a unit from the amount of $1 for a year, or for the interval between the payments, and divide a unit by the remainder; the quotient is the present value of a perpetuity of $1, which multiply by the given perpetuity.

Or, as the given rate : $100 : : the perpetuity : its present value.*

Exam. 3. Required the present value of a house, held on a lease of which 22 years are unexpired, and bringing a profit rent of $450 per annum, payable yearly, compound interest being allowed at 6 per cent. per annum.

Here, the amount of $1 for 22 years is 3.603537. Then dividing 2.603537 by .06, we get 43.3923; the quotient of which by 3.603537 is 12.041583, the present value of an annuity of $1 for 22 years at 6 per cent. per annum. Let this be multiplied by 450, and the product is $5418.71¼, the required value.

Exam. 4. Let every thing be the same as in the last example, except that the annuity is payable *half-yearly*, instead of yearly.

* *The reason* of the first part of this rule is evident from the definition of the present value of an annuity, (page 256,) and from Prob. I. in this article, and Rule III. of compound interest. It might also be shown from Rule III. of compound interest, that at any particular rate, as 5 per cent., the present value of an annuity of $1 is the sum of the decreasing series of continual proportionals, whose terms are the present worths of $1 at compound interest for 1 year, 2 years, 3 years and so on; the first term being 1÷1.05, the ratio 1.05, and the number of terms equal to the number of years. Now, the summation of this series, according to Rule II., (page 249,) agrees exactly with the first part of the rule given above: and the summation of the *infinite* series of present worths, according to the second part of the rule in the text of the same page, agrees with the rule here given for a perpetuity.

In this case, the amount of \$1, at the end of 6 months, is 1.029563, the square root of 1.06; the 44th power of which, (44 being the number of payments,) or its equal, the 22d power of 1.06, is 3.603537. From this take 1, and divide the remainder, 2.603537, by .029563, and the result by 3.603537; the quotient, 24.43916, is the present value of each payment, which being multiplied by \$225, the half-yearly payment, the product is \$5498.81$\frac{1}{10}$, the amount required.

Exam. 5. Required the value of a perpetuity of \$80 a year, at 6 per cent. per annum.

As \$6 : \$100 :: \$80 : \$1333.33$\frac{1}{3}$; or, \$80 ÷ .06 = \$1333.33$\frac{1}{3}$, the value required.

Exam. 6. Suppose the same perpetuity as in the last question, payable half-yearly: what is its present value?

At 6 per cent., the amount of \$1, at the end of half a year, is \$1.029563; and in this question each payment is \$40: therefore, the value of the perpetuity is \$40 ÷ .029563 = \$1353.0426, or \$1353.04, nearly, exceeding that found in the last question, in consequence of the frequency of the payments, by \$19.70$\frac{3}{4}$.

Ex. 3. How much must a person pay to have a salary of \$620 per annum, for 19 years, being allowed compound interest at 5 per cent. per annum? Ans. \$7492.88.

4. Suppose a widow to be entitled to an annuity of \$200, payable half-yearly, from a fund, for 8 years: what is its worth at 6 per cent. per annum, compound interest?

Ans. \$1260.31.

PROBLEM III.—*To find the present value of an annuity in reversion.*

227. RULE.—Find by Prob. II. the present value of the annuity, from the present time till the end of the period of its continuance: find also its value for the time before it comes into possession: the difference of these two results will be the present value required.*

* The reason of this rule is so obvious as to require no explanation. The following rule is also founded on obvious principles, and may, perhaps, be preferred by some:—*Find, by Prob. II., the present value of the annuity during the time it is to be possessed: then, the present value of this result, found by Rule III., compound interest, will be the present value of the reversion.*

Exam. 7. A father leaves to his eldest child for 8 years a profit rent of $280, per annum, payable yearly, and the reversion of it for the 12 years succeeding, to his second child. What is the present value of the legacy of the second, at 4 per cent. per annum, compound interest?

Here, by Prob. II., the value of an annuity of $1 for 20 years, at 4 per cent., is 13.590325, and for 8 years 6.732745; the difference of which is 6.85758, the present value of a reversion of $1 in the proposed circumstances. This being multiplied by $280, the product, $1920.1224, or $1920.12¼, nearly, is the value required.

Ex. 5. What sum must be paid, to change into a perpetuity a lease for 16 years, which brings a profit rent of $286.65 per annum, payable yearly, compound interest being allowed at 4 per cent. per annum? Ans. $3465.34⅙.

6. What sum must be paid, to add 25 years to a lease, which brings a profit rent of $562.50, and of which 14 years are unexpired, compound interest being allowed at 5 per cent. per annum? Ans. $4004.10.

7. What is the present value of the reversion of a perpetuity of $300 per annum, payable yearly, but not to come into possession till the expiration of 100 years, compound interest being allowed at 6 per cent. per annum?

Ans. $14.50¼.

Annuities Contingent, or Life Annuities.

228. *Life Annuities* are those whose commencement or termination, or both, depend on the extinction of one or more lives.

229. When life annuities are in possession, they are often called simply *annuities on lives;* but when they are in reversion, they are generally called *annuities on survivorships.*

230. The *value of a life* is the present value of an annuity of $1 to continue during that life.

231. The *expectation of a life* of a given age, is the mean period during which persons of that age live.

232. The *complement of a life* is double the expectation of the same life.

The calculation of life annuities depends on the joint application of the rules of compound interest, and of the doc-

trine of chances, to tables deduced from observations on the duration of human life. For the theory of life annuities, which is of a nature too complicated to be given in a work like the present, the wishers to be acquainted with this interesting and difficult subject, may have recourse to the writings of *Simpson*, *De Moivre*, and more particularly of *Dr. Price* and *Morgan*, where the subject will be found treated at great length, both in theory and practice. A selection of the most useful rules, without the theory, is given in *Professor Thomson's Arithmetic.* The student is also referred to *Joyce's Arithmetic.*

CHAPTER XVI.

SHORT METHODS OF CALCULATION.

233. An *Aliquot Part* of a number is such a part as, when taken a certain number of times, will exactly make that number. Thus, 5 is an aliquot part of 20, 3 of 12, &c.

234. What is generally called *Practice* is only an abridged method of performing operations in the Rule of Proportion, by the use of aliquot parts; and is generally employed in calculating the prices of commodities. It may be also employed in calculating interest, discount, &c.

TABLES OF ALIQUOT PARTS.

cts.		*ms.*		*s.*	*d.*		*d.*	
50	$= \frac{1}{2}$	5	$= \frac{1}{2}$	10	0	$= \frac{1}{2}$	6	$= \frac{1}{2}$
25	$= \frac{1}{4}$	2	$= \frac{1}{5}$	5	0	$= \frac{1}{4}$	4	$= \frac{1}{3}$
20	$= \frac{1}{5}$	$2\frac{1}{2}$	$= \frac{1}{4}$	4	0	$= \frac{1}{5}$	3	$= \frac{1}{4}$
$12\frac{1}{2}$	$= \frac{1}{8}$	1	$= \frac{1}{10}$	3	4	$= \frac{1}{6}$	2	$= \frac{1}{6}$
$6\frac{1}{4}$	$= \frac{1}{16}$			2	6	$= \frac{1}{8}$	$1\frac{1}{2}$	$= \frac{1}{8}$
5	$= \frac{1}{20}$			2	0	$= \frac{1}{10}$	1	$= \frac{1}{12}$
4	$= \frac{1}{25}$			1	8	$= \frac{1}{12}$	$\frac{3}{4}$	$= \frac{1}{16}$
$2\frac{1}{2}$	$= \frac{1}{40}$			1	4	$= \frac{1}{15}$	$\frac{1}{2}$	$= \frac{1}{24}$
2	$= \frac{1}{50}$			1	3	$= \frac{1}{16}$		
1	$= \frac{1}{100}$			1	0	$= \frac{1}{20}$		
				0	6	$= \frac{1}{40}$		
Of a dollar.		Of a cent.		Of a pound.			Of a shilling.	

Great Hundred.

2 qrs $=\frac{1}{2}$ cwt.	8 lbs. $=\frac{1}{14}$ cwt.	8 lbs. $=\begin{cases}\frac{1}{7} \text{ of } 2 \text{ qrs} \\ \frac{1}{2} \text{ of } 16 \text{ lbs}\end{cases}$
1 qr $=\frac{1}{4}$ cwt.	7 lbs. $=\frac{1}{16}$ cwt.	7 lbs $=\begin{cases}\frac{1}{8} \text{ of } 2 \text{ qrs} \\ \frac{1}{4} \text{ of a qr.}\end{cases}$
16 lbs. $=\frac{1}{7}$ cwt.	4 lbs. $=\frac{1}{28}$ cwt.	4 lbs. $=\begin{cases}\frac{1}{7} \text{ of } 1 \text{ qr.} \\ \frac{1}{4} \text{ of } 16 \text{ lbs}\end{cases}$
14 lbs. $=\frac{1}{8}$ cwt.	14 lbs. $=\begin{cases}\frac{1}{4} \text{ of } 2 \text{ q.} \\ \frac{1}{2} \text{ of a q.}\end{cases}$	2 lbs. $=\begin{cases}\frac{1}{8} \text{ of } 16 \text{ lbs} \\ \frac{1}{4} \text{ of } 8 \text{ lbs.}\end{cases}$

Short Hundred.*

2 qrs. or 50 lbs. $=\frac{1}{2}$ of 100 lbs.	25 lbs. $=\frac{1}{2}$ of 50 lbs.
1 qr. or 25 lbs. $=\frac{1}{4}$ of 100 lbs.	$12\frac{1}{2}$ lbs. $=\begin{cases}\frac{1}{4} \text{ of } 50 \text{ lbs.} \\ \frac{1}{2} \text{ of } 25 \text{ lbs.}\end{cases}$
20 lbs. $=\frac{1}{5}$ of 100 lbs.	
10 lbs. $=\frac{1}{10}$ of 100 lbs.	10 lbs. $=\frac{1}{5}$ of 50 lbs.
	$2\frac{1}{2}$ lbs. $=\frac{1}{4}$ of 10 lbs.

Land Measure.

2 roods $=\frac{1}{2}$ acre	20 poles $=\frac{1}{8}$ acre	10 poles $=\frac{1}{4}$ rood
1 rood $=\frac{1}{4}$ acre	16 poles $=\frac{1}{10}$ acre	8 poles $=\frac{1}{5}$ rood.

These tables may be constructed by Prob. VII. page 132, or rather by dividing $1, £1, 1 acre, &c. by 2, 3, 4, &c. and selecting such of the quotients as are free from fractions.

In the calculation of prices, the quantity of the commodity may be of *one denomination*, or of *more than one;* and accordingly the subject divides itself into two branches, with several varieties, as will appear from the following rules and illustrations.

235. Rule I.—In finding the price of a commodity, *when the price of each article*, as well as the quantity, is *of one denomination*, the product of the given price and of the number of articles, will be the price required.

Example 1. Required the price of 289 cwt. of beef, at $5 per cwt.

* This, in fact, is the weight that is now generally adopted by grocers and merchants throughout the United States.

Here, the price of 289 cwt. at $1 per cwt. is evidently $289; and the price of the same at $5 per cwt. must obviously be 5 times that amount.

289 cwt. at $5 per cwt.

$289=price of 289 cwt. at $1 per cwt.
5

$1445=price of 289 cwt. at $5 per cwt.

Exercises.—1. Required the price of 25 cwt. of sugar, at 11 cents per cwt. Ans. 2·75.

2. Required the price of 125 barrels of flour, at $4 per barrel. Ans. $500.

3. Required the price of 756 yards of superfine black cloth, at £3 per yard. Ans. £2268.

4. Required the price of 26 pieces of linen, at £7 per piece. Ans. £182.

236. Rule II.—*When the price is an aliquot part of a higher denomination*, take a like part of the number of articles, and the result will be the price in the higher denomination.

Exam. 2. What cost 96 lbs. of tea, at 50 cents per lb.?

96 lbs. of tea, at 50 cents per lb.

$96=price of 96 lbs. at $1 per lb.

50cts.=$\frac{1}{2}$ of $1 $48=price of 96 lbs. at 50 cents per lb.

Exam. 3. What cost 532 lbs. of tea, at 6*s*. 8*d*. per lb.?

532 lbs. at 6*s*. 8*d*. per lb.

£532=price of 532 lbs. at £1 each.

6*s*. 8*d*.=$\frac{1}{3}$ of £1 £177 6*s*. 8*d*.=price at 6*s*. 8*d*. each.

In this example, since 532 articles, at £1 each, would cost £532, it is evident, that at 6*s*. 8*d*. the same number of articles would cost one-third of that amount, 6*s*. 8*d*. being one-third of £1. We, therefore, divide 532 by 3, and the quotient £177 6*s*. 8*d*. is the required price.

Ex. 5. What cost 253 lbs. coffee, at 20 cents per lb.? Ans. $50·60.

6. What cost 2560 lbs. cotton, at 12½ cents per lb.? Ans. $320.

7. What cost 139 yards calico, at 25 cents per yard? Ans. $34·75.

8. What cost 335 lbs. vitriol, at 5 cents per lb.? Ans. \$16·75.

9. What cost 676 lbs. of bread, at 4 cents per lb.? Ans. \$27·04.

10. What cost 350 yards of linen, at 3*s.* 4*d.* per yard? Ans. £58 6*s.* 8*d.*

11. What cost 1200 yards of linen, at 2*s.* 6*d.* per yard? Ans. £150.

237. Rule III.—*When the price of each article is not an aliquot part of a higher denomination*, it is to be divided into such parts, that the price of the whole quantity at each of these prices, may be found by the first or second rule; and the sum of the prices thus obtained will be the whole price required.

Exam. 4. Required the price of 479 cwt. of sugar, at \$8·75 per cwt.

	479 cwt. at \$8·75 per cwt.	
	\$479=price of 479 cwt. at \$1 per cwt.	
	8	
	\$3832=	at \$8 per cwt.
50cts.=$\frac{1}{2}$ of 1 cwt.	239·50=	at 50 cents per cwt.
25cts.=$\frac{1}{4}$ of 1 cwt. or 25cts.=$\frac{1}{2}$ of $\frac{1}{2}$do	119·75=	at 25 cents per cwt.
	Ans. \$4191·25=	at \$8·75 per cwt.

Ex. 12. What cost 35 casks of raisins, at \$2·25 per cask? Ans. \$78·75.

13. What cost 120 gallons of wine, at \$2·37$\frac{1}{2}$ per gallon? Ans. \$285.

14. What cost 230 gallons of Madeira wine, at \$2·50 per gallon? Ans. \$575.

15. What cost 356 yards of cloth, at £2 17*s.* 9*d.* per yard? Ans. £1027 19*s.* 0*d.*

238. Rule IV.—*When the quantity is not expressed by a whole number of one denomination*, find the price of the integral quantity according to the method already illustrated, and then find the price of the fractional parts, or lower denominations, from the given rate, by

means of aliquot parts, or otherwise: the sum of all will be the whole price required.

Exam. 5. Required the price of 79 yards 3 qrs. of cloth, at £1 2*s.* 11*d.* per yard?

	79 yds. 3 qrs. at £1 2*s.* 11*d.* per yd.	
	£79=price of 79 yards at £1 per yd.	
2*s.* 6*d.*=£$\frac{1}{8}$	9 17*s.* 6*d.*=	price at 2*s.* 6*d.*
5*d.*=$\frac{1}{6}$ of 2*s.* 6*d.*	1 12*s.* 11*d.*=	at 5*d.*
$\frac{1}{2}$ of £1 2*s.* 11*d.*	11*s.* 5$\frac{1}{2}$*d.*=	of $\frac{1}{2}$ yd.
$\frac{1}{2}$ of 11*s.* 5$\frac{1}{2}$*d.*	5*s.* 8$\frac{3}{4}$*d.*=	$\frac{1}{4}$ yard.
	£91 7*s.* 7$\frac{1}{4}$*d.* Ans.	

In resolving this example, the price of 79 yards is first found, (or rather the parts of which it is made up are found;) and then for half a yard the half of £1 2*s.* 11*d.* is taken, and for a quarter of a yard the half of that is taken: then the sum of all those parts is £91 7*s.* 7$\frac{1}{4}$*d.* the result required.

Exam. 5. What cost 135 acres, 3 roods, 20 poles, at \$5·62$\frac{1}{2}$ per acre?

	135 acrs. 3 r. 20 p. at \$5·62$\frac{1}{2}$ per acre.	
	\$135=the price of 135 acrs. at \$1 per acr.	
	5	
	\$675=	135 at \$5
50cts.=$\frac{1}{2}$ of \$1	67·50=	at 50 cents.
12$\frac{1}{2}$cts.=$\frac{1}{4}$ of \$$\frac{1}{2}$	16·87$\frac{1}{2}$=	at 12$\frac{1}{2}$ cents.
2r.=$\frac{1}{2}$ of an acre	2·81$\frac{1}{4}$=	
1r.=$\frac{1}{2}$ of $\frac{1}{2}$ an acre	1·40$\frac{5}{8}$=	
20p.=$\frac{1}{2}$ of 1 rood	·70$\frac{5}{16}$=	

Ans. \$764·29$\frac{11}{16}$=the price of 135 acres, 3 roods, 20 poles, at \$5·62$\frac{1}{2}$ per acre.

Ex. 16. What cost 236$\frac{3}{4}$ yards of carpeting, at \$1·31$\frac{1}{4}$ per yard? Ans. \$310·73$\frac{1}{2}$.

17. What cost 165 cwt. 3 qrs. 14 lbs. of pork, short weight, at \$5·43$\frac{3}{4}$ per cwt. Ans. \$901·94·8.

18. What is the cost of a farm, containing 256 acres, 1 rood, 30 poles, at \$6·80 per acre? Ans. \$1743·77·5.

19. What cost 105 tons, 17 cwt. 3 qrs. 20 lbs. of sugar, at \$9·60 per cwt.? Ans. \$20332·11·3+.

239. RULE V.—In many calculations, instead of multiplying the quantity by the price, it is better to *multiply the price by the quantity.* This is often the case, when compound multiplication can conveniently be employed.

Exam. 6. Required the price of 12 cwt. 2 qrs. 8 lbs. of hops, at £6 16*s.* 6*d.* per cwt.?

12 cwt. 2 qrs. 8 lbs. at	£6	16	6	
			12	
	£81	18	0	=price of 12 cwt.
2 qrs.=$\frac{1}{2}$ of 1 cwt.	3	8	3	= 2 qrs.
8 lbs.=$\frac{1}{7}$ of 2 qrs.	0	9	9	= 8 lbs.
	£83	16	0	

Ex. 20. Required the price of 18 cwt. 3 qrs. 24 lbs. of hops, at $16·87 per cwt. Ans. $319·92·7+.

21. The quantity of wool exported from Ireland in 1806, was 4337 cwt. 3 qrs. 8 lbs. What was the value at £3 19*s.* 7$\frac{1}{2}$*d.* per cwt. Ans. £17269 18*s.* 10$\frac{1}{2}$*d.*

22. What is the freight of 39 tons, 17 cwt. 3 qrs. of ashes, at £1 17*s.* 6*d.* sterling money per ton, which is the usual freight at present from New-York to Liverpool?

Ans. £74 15*s.* 9$\frac{1}{4}$*d.*

240. RULE I.—*To find the interest of a given sum for any number of days:* multiply the principal, the days, and twice the rate, continually together; and divide the product by 73000.

Exam. 7. What is the interest of $372.50, from February 12, till December 17, 1827, at 4$\frac{1}{2}$ per cent. per annum?

Here, $372.50×308, (the number of days from February 12, till December 17,) =$114730.00; this multiplied by 9, (the double of the rate,) =1032570.00; and $1032570.00 ÷73000=$14.14$\frac{1}{2}$, the answer.*

* *The reason of the rule* will be evident from the operation in compound proportion, if instead of $100 and the rate per cent. their doubles be employed. Thus, we should have in this example,

$$\left.\begin{matrix}\text{As } \$100 : \$9 \\ 365 \text{ days} : 308 \text{ days}\end{matrix}\right\} :: \$372.50 : \$14.14\tfrac{1}{2};$$

and in working this, we should, by the rule for compound proportion, multiply together the principal, the days, and twice the rate, and divide the product by 365×200, or 73000.

241. *When the rate is 5 per cent.*, divide the product of the principal and days by 7300; *for other rates than 5 per cent.*, increase or diminish the product of the principal and days, by the method of aliquot parts, and then proceed by the rule.*

Exam. 8. Required the interest of \$250 for 63 days, at 6 per cent. per annum.

Here, $\$250\times63=\$15750$, to which add one-fifth of itself, and the sum will be \$18900; then, $\$18900\div7300=\$2.59$, nearly, the interest required.

242. Rule II.—*To find the discount of a given sum for 60 days, at 6 per cent., as practised in the banks of New-York:* divide the given sum by 100, and the quotient will be the discount required. For any other number of days than 60, increase or diminish the given sum, by the method of aliquot parts, and then proceed by the rule.

Exam. 9. What is the discount on a note of \$575, that has 60 days to run, at 6 per cent. per annum?

Here, $\$575\div100=\$5.75$, the discount.†

Ex. 23. What is the discount on a note of \$675, for 63 days, at 6 per cent. per annum? Ans. \$7.08.7½.

24. What is the discount on a note for \$1150.75, for 90 days, at 6 per cent. per annum? Ans. \$17.26.1¼.

The following is another *false* rule, that is generally used in computing discount on notes.

243. Rule III.—*To find the bank discount on notes that have 33, 63, or any number of days to run, inclu-*

* The reason of this rule is obvious from the last, since the double of 5 is 10, and $73000\div10=7300$.

† *The reason of this rule* is obvious from the operation in compound proportion, and from this *false* principle, the year being reckoned only 360 days: thus,

$$\left.\begin{array}{l}\text{As } \$100 : \$6 \\ 360 \text{ days} : 60 \text{ days}\end{array}\right\} :: \$575 : \$5.75;$$

$$\text{Or, as } \$100 : \$1 :: \$575 : \$5.75.$$

If it had 63 days to run, add the one-twentieth of \$5.75 to itself, and the sum, \$6.03¾, will be the discount for 63 days; this is obvious, since 3 days is the one-twentieth of 60 days.

ding 3 days of grace: multiply the amount of the note by the number of days it has to run, and divide that product by 6000; the quotient will be the discount required.

Exam. 10. What is the discount on a note of \$500, for 33 days, at 6 per cent. per annum?

Here, \$500×33=\$16500; then, \$16500÷6000=\$2.75, the discount required.*

Ex. 25. What is the discount of \$9760, for 63 days, at 6 per cent. per annum? Ans. \$102.48.

26. What is the discount of \$870.75, for 93 days, at 6 per cent. per annum? Ans. \$13.49.6$\frac{5}{8}$.

244. RULE IV.—*To find the interest of a given sum for any number of days.* Multiply the principal, the days, the rate, and 274, continually together; reject from the resulting product four figures to the right, and the remainder will be the interest in mills.

If the principal contain dollars and cents, six figures must be rejected from the product.

Exam. 11. Find the interest of \$619 for 126 days, at 5 per cent. per annum.

Here, 619×126×5×274=106851780; then, rejecting four figures to the right, we have 10685 mills, or \$10.68$\frac{1}{2}$, the interest required.†

Ex. 27. Find the interest of \$754.87 for 147 days, at 6 per cent. per annum. Ans. \$18.24.3, nearly.

* *The reason of this rule* depends upon the same principle as the last: that is,

$$\left.\begin{array}{l}\text{As } \$100 : \$6 \\ 360 \text{ days} : 33 \text{ days}\end{array}\right\} :: \$500 : \$2.75;$$

Or, as 6000 : 33 : : \$500 : \$2.75, since 360×100÷6=6000.

† *The reason of this rule* is evident from compound proportion and decimals. Thus, in this example, we should have,

$$\left.\begin{array}{l}\text{As } \$100 : \$5 \\ 365 \text{ days} : 126 \text{ days}\end{array}\right\} :: \$619 : \$10.68\tfrac{1}{2};$$

Here, instead of dividing the continual product of 619, 126, and 5, according to the rule of compound proportion, we multiply it by .0000274, which is found by dividing 1 by 36500; or, which is the same thing, by multiplying the product by 274, and rejecting four figures, the remainder will be mills. When the time and principal are not very great, this rule will be found extremely near the truth; in large amounts, it may be corrected by deducting one cent for every \$100 in the answer.

TABLE I. SHOWING THE AMOUNT OF $1, AT COMPOUND INTEREST.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·030,000	1·040,000	1·050,000	1·060,000
2	1·060,900	1·081,600	1·102,500	1·123,600
3	1·092,727	1·124,864	1·157,625	1·191,016
4	1·125,509	1·169,859	1·215,506	1·262,477
5	1·159,274	1·216,653	1·276,282	1·338,226
6	1·194,052	1·265,319	1·340,096	1·418,519
7	1·229,874	1·315,932	1·407,100	1·503,630
8	1·266,770	1·368,569	1·477,455	1·593,848
9	1·304,773	1·423,312	1·551,328	1·689,479
10	1·343,916	1·480,244	1·628,895	1·790,848
11	1·384,234	1·539,454	1·710,339	1·898,299
12	1·425,761	1·601,032	1·795,856	2·012,196
13	1·468,534	1·665,074	1·885,649	2·132,928
14	1·512,590	1·731,676	1·979,932	2·260,904
15	1·557,967	1·800,944	2·078,928	2·396,558
16	1·604,706	1·872,981	2·182,875	2·540,352
17	1·652,848	1·947,900	2·292,018	2·692,773
18	1·702,433	2·025,817	2·406,619	2·854,339
19	1·753,506	2·106,849	2·526,950	3·025,600
20	1·806,111	2·191,123	2·653,298	3·207,135
21	1·860,295	2·278,768	2·785,963	3·399,564
22	1·916,103	2·369,919	2·925,261	3·603,537
23	1·973,587	2·464,716	3·071,524	3·819,750
24	2·032,794	2·563,304	3·225,100	4·048,935
25	2·093,778	2·665,836	3.386,355	4·291,871
26	2·156,592	2·772,470	3·555,673	4·549,383
27	2·221,289	2·883,369	3·733,456	4·822,346
28	2·287,928	2·998,703	3·920,129	5·111,687
29	2·356,566	3·118,651	4·116,136	5·418,388
30	2·427,262	3·243,398	4·321,942	5·743,491
31	2·500,080	3·373,133	4·538,039	6·088,101
32	2·575,083	3·508,059	4·764,941	6·453,386
33	2·652,335	3·648,381	5·003,189	6·840,590
34	2·731,905	3·794,316	5·253,348	7·251,025
35	2·813,862	3·946,089	5·516,015	7·686,087
36	2·898,278	4·103,933	5·791,816	8·147,252
37	2·985,227	4·268,090	6·081,407	8·636,087
38	3·074,783	4·438,813	6·385,477	9·154,252
39	3·167,027	4·616,366	6·704,751	9·703,507
40	3·262,038	4·801,021	7·039,989	10·285,718
41	3·359,899	4·993,061	7·391,988	10·902,861
42	3·460,696	5·192,784	7·761,588	11·557,033
43	3·564,517	5·400,495	8·149,667	12·250,455
44	3·671,452	5·616,515	8·557,150	12·985,482
45	3·781,596	5·841,176	8·985,008	13·764,611
46	3·895,044	6·074,823	9·434,258	14·590,487
47	4·011,895	6·317,816	9·905,971	15·465,917
48	4·132,252	6·570,528	10·401,270	16·393,872
49	4·256,219	6·833,349	10·921,333	17·377,504
50	4·383,906	7·106,683	11·467,400	18·420,154

TABLE II.—SHOWING THE AMOUNT OF AN ANNUITY OF $1.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·000,000	1·000,000	1·000,000	1·000,000
2	2·030,000	2·040,000	2·050,000	2·060,000
3	3·090,900	3·121,600	3·152,500	3·183,600
4	4·183,627	4·246,464	4·310,125	4·374,616
5	5·309,135	5·416,322	5·525,631	5·637,092
6	6·468,409	6·632,975	6·801,912	6·975,318
7	7·662,462	7·898,294	8·142,008	8·393,837
8	8·892,336	9·214,226	9·549,108	9·897,467
9	10·159,106	10·582,795	11·026,564	11·491,315
10	11·463,879	12·006,107	12·577,892	13·180,794
11	12·807,795	13·486,351	14·206,787	14·791,642
12	14·192,029	15·025,805	15·917,126	16·869,941
13	15·617,790	16·626,837	17·712,982	18·882,137
14	17·086,324	18·291,911	19·598,631	21·015,065
15	18·598,913	20·023,587	21·578,563	23·275,969
16	20·156,881	21·824,531	23·657,491	25·672,528
17	21·761,587	23·697,512	25·840,366	28·212,879
18	23·414,435	25·645,412	28·132,384	30·905,652
19	25·116,868	27·671,229	30·539,003	33·759,991
20	26·870,374	29·778,078	33·065,954	36·785,591
21	28·676,485	31 969,201	35·719,251	39·992,726
22	30·536,780	34·247,969	38·505,214	43·392,290
23	32·452,883	36·617,888	41·430,475	46·995,827
24	34·426,470	39·082,604	44·501,998	50·815,577
25	36·459,264	41·645,908	47·727,098	54·864,512
26	38·553,042	44·311,744	51·113,453	59·156,382
27	40·709,633	47·084,214	54·669,126	63·705,765
28	42·930,922	49·967,582	58·402,582	68·528,111
29	45·218,850	52·966,286	62·322,711	73·639,798
30	47·575,415	56·084,937	66·438,847	79·058,186
31	50·002,678	59·328,335	70·760,789	84·801,677
32	52·502,758	62·701,468	75·298,829	90·889,778
33	55·077,841	66·209,527	80·063,770	97·343,164
34	57·730,176	69·857,908	85·066,959	104·183,754
35	60·462,081	73·652,224	90·320,307	111·434,779
36	63·275,944	77·598,313	95·836,322	119·120,866
37	66·174,222	81·702,246	101·628,138	127·268,118
38	69·159,449	85·970,336	107·709,545	135·904,205
39	72·234,232	90·409,149	114·095,023	145·058,458
40	75·401,259	95·025,515	120·799,774	154·761,965
41	78·663,297	99·826,536	127·839,762	165·047,683
42	82·023,196	104·819,597	135·231,751	175·950,544
43	85·483,892	110·012,381	142·993,338	187·507,577
44	89·048,409	115·412,876	151·143,005	199·758,031
45	92·719,861	121·029,392	159 700,155	212·743,513
46	96·501,457	126·870,567	168·685,163	226·508,124
47	100·396,500	132·945,390	178·119,421	241·098,612
48	104·408,395	139·263,206	188·025,392	256·564,528
49	108·540,647	145·833,734	198·426,662	272·958,400
50	112·796,867	152·667,083	209·347,995	290·335,904